SE 504 (Formal Methods and Models)
Spring 2025
HW #1: Predicate Strength/Weakness and Hoare Triple Laws
Due: 4pm, Friday, Feb 7

For each of Problems 1 through 7, indicate the weakness/strength relationship that exists between the two given predicates, P and Q. Recall that there are four possibilities: P and Q are equivalent, P is strictly stronger than Q, P is strictly weaker than Q, or none of the above. For a more detailed treatment, follow the *On the Strength/Weakness Relationship between Predicates* link on the course web page.) For at least one problem, the theorems in *On Proofs Involving the Replacement of A by B, where A implies B* will be useful. Specifically, recall that weakening (respectively, strengthening) the antecedant of an implication strengthens (respectively, weakens) the implication as a whole. Meanwhile, weakening (respectively, strengthening) the consequent of an implication weakens (respectively, strengthens) the implication as a whole.

You must justify your answers, but you need not provide formal justifications for "obvious" theorems of arithmetic, such as $x > y \Rightarrow x \geq y$, or $x \geq y + 4 \Rightarrow x \geq y$, or $x < y \Rightarrow x \neq y$. Just cite "number theory" or "arithmetic".

To show $[P \Rightarrow Q]$ (i.e., "$P \Rightarrow Q$ holds in all states"), it suffices to prove $P \Rightarrow Q$. To show $\neg[P \Rightarrow Q]$, one should identify a counterexample to $P \Rightarrow Q$ (i.e., by identifying one or more states in which P is true but Q is false).

1. $P : x \geq 2 \ \wedge \ y \geq x + 3$ and $Q : x > 0 \ \wedge \ y > x$

2. $P : x \geq 2 \ \vee \ y > x + 3$ and $Q : x > 0 \ \wedge \ y > x$

3. $P : x \geq 0 \ \vee \ y < x$ and $Q : x > -2$

4. $P : x > -2$ and $Q : x > 0 \ \wedge \ y < x$

5. $P : x > 0 \ \wedge \ y \neq x$ and $Q : x = 5$

6. $P : x \geq 0 \ \Rightarrow \ y < z$ and $Q : x = 4 \ \Rightarrow \ y \leq z$

7. $P : f.k = 1$ and $Q : (\exists k \mid : f.k = 1)$

For the next two problems, use one or more of the **Strengthening the Precondition**, **Weakening the Postcondition**, **Precondition Disjunctivity**, and **Postcondition Conjunctivity** Laws of Hoare Triples (a link to which you can find on the course web page), as well as "obvious" theorems of arithmetic and theorems from Gries and Schneider, to prove the stated implications.

8. $[Q_0 \Rightarrow Q_1] \ \Longrightarrow \ (\{P\}\ S\ \{Q_0\} \wedge \{P\}\ S\ \{Q_1\} \ \equiv \ \{P\}\ S\ \{Q_0\})$

Hint: Assume the antecedant and prove the consequent.

9. If $\{x \geq y\}\ S\ \{x > 5y \ \wedge \ y > z\}$ and $\{y < 7\}\ S\ \{x > 5y \ \wedge \ y < z\}$, then
$\{x > y \ \vee \ y \leq 4\}\ S\ \{x \geq 5y \ \wedge \ y \neq z\}$

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For the last two problems, identify both the weakest Y and the strongest Y satisfying the given "equation". In developing your answers, it may be helpful to think in terms of *satisfying state sets* rather than predicates and to use these facts:

$$\begin{aligned} \widehat{(P \wedge Q)} &= \hat{P} \cap \hat{Q} \\ \widehat{(P \vee Q)} &= \hat{P} \cup \hat{Q} \\ [P \Rightarrow Q] &\equiv \hat{P} \subseteq \hat{Q} \end{aligned}$$

Recall that, where R denotes a predicate, $\hat{R}$ denotes the set containing precisely those states that satisfy R (i.e., in which R evaluates to true).

10. $Y : \ [P \wedge Y \ \Rightarrow \ Q]$.

11. $Y : \ [P \vee Q \ \Rightarrow \ P \vee Y]$.