We showed above that $R[x, y := \ldots]$ is, in general, different from $R[x := \ldots][y := \ldots]$, so it should be no surprise that these two assignments have different effects.

It is a shame that the multiple assignment is not included in more programming languages. The programmer is frequently called upon to specify a state change that involves modifying several variables in one step, where the values assigned all depend on the initial state, and the multiple assignment is ideally suited for this task.

Exercises for Chapter 1

1.1 Perform the following textual substitutions. Be careful with parenthesization and remove unnecessary parentheses.

(a) $x[x := b + 2]$
(b) $x + y \cdot x[x := b + 2]$
(c) $(x + y \cdot x)[x := b + 2]$
(d) $(x + x \cdot 2)[x := x \cdot y]$
(e) $(x + x \cdot 2)[y := x \cdot y]$
(f) $(x + x \cdot y + x \cdot y \cdot z)[x := x + y]$

1.2 Perform the following simultaneous textual substitutions. Be careful with parenthesization and remove unnecessary parentheses.

(a) $x[x, y := b + 2, x + 2]$
(b) $x + y \cdot x[x, y := b + 2, x + 2]$
(c) $(x + y \cdot x)[x, y := b + 2, x + 2]$
(d) $(x + x \cdot 2)[x, y := x \cdot y, x \cdot y]$
(e) $(x + y \cdot 2)[y, x := x \cdot y, x \cdot x]$
(f) $(x + x \cdot y + x \cdot y \cdot z)[x, y := y, x]$

1.3 Perform the following textual substitutions. Be careful with parenthesization and remove unnecessary parentheses.

(a) $x[x := y + 2][y := y \cdot x]$
(b) $x + y \cdot x[x := y + 2][y := y \cdot x]$
(c) $(x + y \cdot x)[x := y + 2][y := y \cdot x]$
(d) $(x + x \cdot 2)[x, y := y, x][x := z]$
(e) $(x + x \cdot 2)[x, y := x, z][x := y]$
(f) $(x + x \cdot y + x \cdot y \cdot z)[x, y := y, x][y := 2 \cdot y]$

1.4 Leibniz's definition of equality given just before inference rule Leibniz (1.5) says that $X = Y$ is true in every state iff $E[z := X] = E[z := Y]$ is true in every state. Inference rule Leibniz (1.5), however, gives only the "if" part. Give an argument to show that the "only if" part follows from Leibniz (1.5). That is, suppose $E[z := X] = E[z := Y]$ is true in every state, for every expression E. Show that $X = Y$ is true in every state.

1.5 Let X , Y , and Z be expressions and z a variable. Let E be an expression, which may or may not contain Z . Here is another version of Leibniz.

$$\textbf{Leibniz:}\quad \frac{Z = X,\ Z = Y}{E[z := X] = E[z := Y]} \ .$$

Show that transitivity of $=$ follows from this definition.

1.6 Inference rule Substitution (1.1) stands for an infinite number of inference rules, each of which is constructed by instantiating expression E , list of variables v , and list of expressions F with different expressions and variables. Show three different instantiations of the inference rule, where E is $x < y \vee x \geq y$.

1.7 Inference rule Leibniz (1.5) stands for an infinite number of inference rules, each of which is constructed by instantiating E, X, and Y with different expressions. Below, are a number of instantiations of Leibniz, with parts missing. Fill in the missing parts and write down what expression E is. Do not simplify. The last two exercises have three answers; give them all.

(a) $\dfrac{x = x + 2}{4 \cdot x + y = \ ?}$

(b) $\dfrac{2 \cdot y + 1 = 5}{x + (2 \cdot y + 1) \cdot w = \ ?}$

(c) $\dfrac{x + 1 = y}{3 \cdot (x + 1) + 3 \cdot x + 1 = \ ?}$

(d) $\dfrac{x = y}{x + x = \ ?}$

(e) $\dfrac{7 = y + 1}{7 \cdot x + 7 \cdot y = \ ?}$

1.8 The purpose of this exercise is to reinforce your understanding of the use of Leibniz (1.5) along with a hint in proving two expressions equal. For each of the expressions $E[z := X]$ and hints $X = Y$ below, write the resulting expression $E[z := Y]$. There may be more than one correct answer.

	$E[z := X]$	hint $X = Y$
(a)	$x + y + w$	$x = b + c$
(b)	$x + y + w$	$b \cdot c = y + w$
(c)	$x \cdot (x + y)$	$x + y = y + x$
(d)	$(x + y) \cdot w$	$w = x \cdot y$
(e)	$(x + y) \cdot q \cdot (x + y)$	$y + x = x + y$

1.9 The purpose of this exercise is to reinforce your understanding of the use of Leibniz (1.5) along with a hint in proving two expressions equal. For each of the following pair of expressions $E[z := X]$ and $E[z := Y]$, identify a hint $X = Y$ that would show them to be equal and indicate what E is.

	$E[z := X]$	$E[z := Y]$
(a)	$(x+y)\cdot(x+y)$	$(x+y)\cdot(y+x)$
(b)	$(x+y)\cdot(x+y)$	$(y+x)\cdot(y+x)$
(c)	$x+y+w+x$	$x+y\cdot w+x$
(d)	$x\cdot y\cdot x$	$(y+w)\cdot y\cdot x$
(e)	$x\cdot y\cdot x$	$y\cdot x\cdot x$

1.10 In Sec. 1.3, we stated that the four laws Reflexivity (1.2), Symmetry (1.3), Transitivity (1.4), and Leibniz (1.5) characterized equality. This statement is almost true. View $=$ as a function $eq(x,y)$ that yields a value *true* or *false*. There is one other function that, if used in place of eq in the four laws, satisfies all of them. What is it?

1.11 Using Definition (1.12) of the assignment statement on page 18, determine preconditions for the following statements and postconditions.

	Statement	Postcondition
(a)	$x := x+7$	$x+y>20$
(b)	$x := x-1$	$x^2+2\cdot x=3$
(c)	$x := x-1$	$(x+1)\cdot(x-1)=0$
(d)	$y := x+y$	$y=x$
(e)	$y := x+y$	$y=x+y$