4. CAPITAL BUDGETING UNDER CERTAINTY

Objectives: After reading this chapter, you will
1. Understand the concept of net present value.
2. Use *NPV* approach in decision-making.
3. Understand the meaning of internal rate of return. Appreciate the difficulty in applying this concept, and its inability to give a unique, or optimal solution.
4. See the effect of depreciation and taxes on the investment decisions.

4.1 Video 04A Capital Budgeting

In the course of their business, firms have to make capital investment decisions. This involves critical evaluation of long-term investments and their impact on the value of the company. The corporations make large investments in buildings and land, and in plant and equipment. There is a constant need for modernization of equipment due to changes in technology. As the firm grows, they need larger facilities. A firm may embark on new projects, which may entail large investment of time and capital. The firm considers all these decisions in light of the long-term benefit of the corporation. From the financial point of view, only those projects will be acceptable that add to the value of the firm, and increase the wealth of the owners of the firm.

A company has to evaluate many projects. Some of these projects may be mutually exclusive in the sense that you have to pick only one and exclude others. A company may want to install gas heat, or oil heat, in a factory, but not both. The company may have to evaluate several alternative projects and rank them according to their profitability. Finally, they may have to pick only one or two projects that they can finance with the available capital. Thus, capital budgeting becomes an important issue.

We will consider one of the most important concepts in finance, the *net present value*, which is the optimal decision making model to screen out the profitable projects from the unprofitable ones. The net present value, *NPV*, of a project is the discounted sum of all cash flows of a project, negative and positive, present and future. We may treat the initial investment as a negative cash flow at present. Discount the future cash flows at a rate that depends on the cost of capital of the firm and the risk of the project. By definition,

$$NPV = -I_0 + \sum_{i=1}^{n} \frac{C}{(1+r)^i} \qquad (4.1)$$

In the above expression, we define the symbols as follows:

I_0 = initial investment in the project,
C = after-tax annual earnings of the project,
n = life of the project in years, and
r = risk-adjusted discount rate for the cash flows.

The decision rule in using *NPV* is that if the *NPV* is positive the project is acceptable, otherwise not. This is also in concert with the fundamental aim of the corporation to maximize its value. Alternative decision rules such as the internal rate of return, *IRR* or payback period are inadequate in many situations and may give misleading results.

The payback period method is really quite easy to apply. For example if a firm spends $5000 to start a project that generates an income of $1000 annually, then the payback period is 5 years. On the other hand, a project that costs $8,000 and generates $2000 annually will have the payback period 4 years. Based on the criterion of payback period, the second project with the shorter time is better.

There are three serious defects in the payback period method. The main problem with this methodology is that this procedure ignores the time value of money. It does not discount the future cash flows and treats them at par with the present investment. This violates a very fundamental concept in finance.

The second problem is that we are not looking at the risk of the project. The riskier cash flows should have less value than more secure cash flows. We should adjust the discount rate according the risk involved.

The third drawback is that we are not looking at the entire set of cash flows, meaning, we ignore the cash flows that occur after the time when we have recovered the initial investment. Perhaps there are large negative cash flows that appear after the recovery of the initial investment. This could change the calculation completely. We shall ignore this method of project evaluation completely.

Closely related to the *NPV* method is the internal rate of return method. The internal rate of return method has some merit. Actually, it is merely an extension of the *NPV* method and we shall look at it in the next section.

We shall first consider simple problems in capital budgeting where the cash flows and other outcomes are known with certainty. Later we shall include the complications due to non-uniform cash flows, taxes and depreciation, and resale value. In the next chapter, we shall continue the discussion of capital budgeting under uncertainty.

Examples

4.1. An investment requires the following cash outlays: $10,000 now and $5,000 a year from now. The investment will give a cash return of $5,000 annually for 6 years, the first payment coming in after 3 years. The risk-free rate is 6%. If the proper discount rate is 12%, would you accept this investment?

The firm should look at first two cash flows, $10,000 now, and $5000 a year from now, as definite commitment to finance the project. The firm can certainly pay $5000 next year by investing in a risk-free bond now, whose present value is 5000/1.06. Considering all the cash flows,

$$NPV = -10{,}000 - \frac{5000}{1.06} + \frac{5000}{1.12^3} + \frac{5000}{1.12^4} + \frac{5000}{1.12^5} + \frac{5000}{1.12^6} + \frac{5000}{1.12^7} + \frac{5000}{1.12^8}$$

We can either add them up separately, or combine them with the help of a formula. We may write the above cash flows as

$$NPV = -10{,}000 - \frac{5000}{1.06} + \frac{1}{1.12^2}\left[\frac{5000}{1.12} + \frac{5000}{1.12^2} + \frac{5000}{1.12^3} + \frac{5000}{1.12^4} + \frac{5000}{1.12^5} + \frac{5000}{1.12^6}\right]$$

$$NPV = -10{,}000 - \frac{5000}{1.06} + \frac{1}{1.12^2}\sum_{i=1}^{6}\frac{5000}{1.12^i}$$

Use,

$$\sum_{i=1}^{n}\frac{C}{(1+r)^i} = \frac{C[1-(1+r)^{-n}]}{r} \qquad (2.6)$$

$$NPV = -\ 10{,}000 - \frac{5000}{1.06} + \frac{1}{1.12^2}\left[\frac{5000(1-1.12^{-6})}{.12}\right] = \$1671$$

Because of the positive *NPV*, we should accept the investment. ♥

To solve the problem using WolframAlpha, write the above equation as

WRA `-10000-5000/1.06+sum(5000/1.12^i,i=3..8)`

4.2. A young woman buys a life insurance policy on her 21st birthday. She has to pay an annual premium of $147 through her 64th birthday. On her 65th birthday, she will receive $10,000 as the surrender value of the policy. If she lives long enough to collect herself, and assuming a discount rate of 12% in this case, find the *NPV* of this policy to the owner of the policy.

Of course, the life insurance policy will also provide a $10,000 benefit to her heirs in case she dies before reaching her 65th birthday. Here we are concerned only with the *NPV* of her investment in case she lives to collect the benefits herself. She makes 44 payments of $147 each, the first one right now. She also receives one payment of $10,000 after 44 years. Considering the present value of all the payments, we have

$$NPV = -147 - \sum_{i=1}^{43}\frac{147}{1.12^i} + \frac{10{,}000}{1.12^{44}} = -\$1294.33$$

The negative *NPV* in this case does not mean that she should not buy the insurance. In fact, it may be quite reasonable to provide $10,000 benefits to her children in case she dies before she reaches the age of 65 by paying $1294 in current dollars. ♥

To solve the problem at WolframAlpha, use the following expression

WRA **-sum(147/1.12^i,i=0..43)+10000/1.12^44**

4.3. Devon Inc. wishes to invest \$50,000 in a new project, which will give a return of \$10,000 annually for the first 5 years, and then an uncertain amount every year for the next 5 years. The proper discount rate is 11% annually. Calculate the minimum value of the uncertain return, which will make the project worthwhile for Devon.

Suppose the uncertain cash flow is x. To break even, the *NPV* of the project is zero. Thus, we may write the problem as follows:

$$NPV = 0 = -50{,}000 + \sum_{i=1}^{5} \frac{10{,}000}{1.11^i} + \sum_{i=6}^{10} \frac{x}{1.11^i}$$

The second summation on the right side is equivalent to $\frac{1}{1.11^5}\sum_{i=1}^{5}\frac{x}{1.11^i}$.

Using (2.6), we get

$$50{,}000 - \frac{10{,}000(1-1.11^{-5})}{.11} = \frac{1}{1.11^5}\left(\frac{x(1-1.11^{-5})}{.11}\right)$$

Solving for x, we get

$$1.11^5\left(\frac{.11}{1-1.11^{-5}}\right)\left[50{,}000 - \frac{10\,000(1-1.11^{-5})}{.11}\right] = x$$

This gives $x = \$5945.75$ ♥

To solve the problem at WolframAlpha, use the following expression

WRA **0=-50000+sum(10000/1.11^i,i=1..5)+sum(x/1.11^i,i=6..10)**

4.2 Video 04B Internal Rate of Return

The internal rate of return, or *IRR*, of a project is that particular discount r, which will make the net present value of the project equal to zero. If we let

$$NPV = 0 = -I_0 + \sum_{i=1}^{n} \frac{C}{(1+r)^i} \tag{4.2}$$

and solve the equation for r, then this particular discount rate is the internal rate of return. Once we find the *IRR*, it is compared with the risk-adjusted discount rate for the given project. If *IRR* is greater, the project is accepted.

This equation is difficult to solve in general. However, certain calculators, such as HP-12C, have the capability of getting the answer. If the cash flows are uniform, using tables and interpolating the value of the discount rate may solve the problem. The best way to handle (4.2) is to use WolframAlpha.

Although many managers use the *IRR* as a decision-making tool to accept or reject a project, it has some serious flaws.

First, a project may not have a unique *IRR*. This is because a quadratic or a higher degree equation has multiple roots. Some of these values do not have any economic significance whatsoever, and it is not always possible to identify the correct value. Second, one cannot use the *IRR* method reliably to rank projects. This is again due to multiplicity of roots of the equation.

All these problems are absent in the *NPV* method, which is the optimal method for decision-making. The only advantage of using *IRR* is that one can compare it directly to a hurdle rate, or a minimum acceptable rate of return set by the managers of a corporation.

Examples

4.4. Betsey Trotwood is planning to open a restaurant. Her initial investment will be \$50,000. She expects to receive \$20,000 at the end of first, second, and third year. Find the internal rate of return of her project.

In general, we can solve the internal rate of return problems using Maple. We equate the present value of all cash flows to zero, and find the proper discount rate. In this case,

$$-50{,}000 + \sum_{i=1}^{3} \frac{20{,}000}{(1+r)^i} = 0$$

To solve the problem using WolframAlpha, write the above equation as

WRA **-50000+sum(20000/(1+r)^i,i=1..3)=0**

The real solution is $r \approx .0970103$, which is about 9.7%.

To get the result using Excel, set up the spreadsheet as follows. Adjust the value of the number in B1 until the result in cell B2 becomes very close to 0.

	A	B
1	IRR =	.09701
2	NPV = 0	-50000+20000*(1-1/(1+B1)^3)/B1

4.5. An investment of \$10,000 will return \$3,000 at the end of each of the next five years. Find the *IRR* of this investment.

To solve for *IRR* we set the *NPV* equal to zero. Thus

$$NPV = -10{,}000 + \sum_{i=1}^{5} \frac{3000}{(1+r)^i} = 0$$

To solve the problem using WolframAlpha, write the above equation as

WRA **-10000+sum(3000/(1+r)^i,i=1..5)=0**

This gives the real solution as $r \approx .152382$, which is 15.24%. ♥

4.6. An investment has an initial outlay of $1200; an income of $5,000 at the end of one year; and an expense of $3,000 at the end of second year. Find the internal rate of return of this investment.

By definition, the internal rate of return of an investment is the discount rate that will make its net present value to be zero. Suppose the required discount rate is r. Then

$$NPV = -1200 + \frac{5000}{1+r} - \frac{3000}{(1+r)^2} = 0 \qquad \textbf{(A)}$$

Let $1 + r = x$. Then

$$-1200 + \frac{5000}{x} - \frac{3000}{x^2} = 0$$

or, multiplying by x^2, $\quad -1200\,x^2 + 5000x - 3000 = 0$

Dividing throughout by –200, we get

$$6x^2 - 25x + 15 = 0$$

This is a quadratic equation and we may solve it by using the standard formula. The solution of

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case the solution is

$$x = \frac{25 \pm \sqrt{625 - 4(6)(15)}}{12} = \frac{25 \pm \sqrt{265}}{12} = 3.44 \text{ or } 0.727$$

Since $x = 1 + r$, $r = 2.44, -0.273$. This is a case of *multiple* internal rates of return. There is not much economic sense in the two values of *IRR* calculated above. Therefore, we are unable to decide the case on the basis of *IRR*. ♥

At WolframAlpha, write equation (A) as follows and click on **Approximate forms**.

WRA **-1200+5000/(1+r)-3000/(1+r)^2=0**

Fig. 4.1 shows the calculated value of *NPV* at different discount rates. When the discount rate is zero, *NPV* = \$800. Note that the *NPV* = 0 for $r = -.273$ and 2.44. By differentiating the function

$$-1200 + \frac{5000}{1+r} - \frac{3000}{(1+r)^2}$$

with respect to r and setting the derivative equal to zero, we get the maximum value of *NPV* as \$883 when $r = 20\%$. We can observe that in Fig. 4.1.

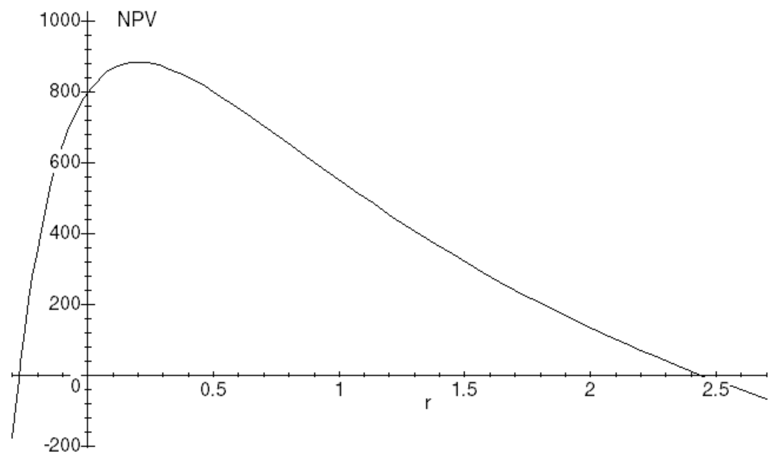

Fig. 4.1: The diagram shows the IRR for a project. The curve crosses the x-axis at $r = -.273$ and $r = 2.44$.

4.7. (A) Jefferson Corporation is considering a project that requires a cash outlay of \$4,000 now, and another \$3,000 expense one year from now. The risk-free rate is 6%. The project will terminate after two years, at which time it will generate a single positive cash flow of \$10,000. Calculate the internal rate of return of this project.

(B) If the cost of capital for Jefferson is 20%, should it undertake the above project?

(C) Verify your answer to (B) by calculating the *NPV* of the project.

(A) Setting the *NPV* equal to zero, we have

$$-4000 - \frac{3000}{1.06} + \frac{10{,}000}{(1+r)^2} = 0$$

Or, $$\frac{10{,}000}{(1+r)^2} = 4000 + \frac{3000}{1.06} = 6830$$

Or, $$(1+r)^2 = 10{,}000/6830 = 1.464$$

Or, $$1 + r = 1.209995206$$

Which gives $$r = .21 = 21\% ♥$$

(B) With the cost of capital at 20%, which is less than the *IRR*, the project is acceptable.♥

(C) $NPV = -4{,}000 - \frac{3000}{1.06} + \frac{10{,}000}{1.2^2} = \$114.26$

Since the *NPV* is positive, the project is indeed acceptable. ♥

4.3 Video 04D Taxes and Depreciation

At this point in our calculation of the net present value of a project, we must also include two important considerations: *depreciation* and *taxes*. They have a strong impact on our decision making process. We also have to contend with them in real life situations.

We know of the physical depreciation; that machinery and equipment wears down with age. The value of old equipment decreases with time. The old equipment is subject to frequent breakdowns and is not quite that productive as new equipment. Modern technology tends to get obsolete rather quickly, that is, depreciates more rapidly. This loss of value is the basis for depreciation as an accounting term.

Depreciation is a non-cash expense and companies use it to offset taxable income. One can calculate the depreciation on a *straight-line basis*. A machine with a 5-year life will have depreciation equal to 20% of its value in each year.

A faster method of depreciation is the sum-of-years-digits method. Since 1 + 2 + 3 + 4 + 5 = 15, the depreciation in the five years will be 5/15, 4/15, 3/15, 2/15 and 1/15, respectively. A third method is the modified accelerated cost recovery system, or MACRS. The following table gives a simplified version of MACRS for assets with a 3-year or 5-year life.

Year	MACRS, 3 year	MACRS, 5 year
1	33.33%	20.00%
2	44.44	32.00
3	14.82	19.20
4	7.41	11.52
5		11.52
6		5.76

What is the optimal depreciation policy of a corporation? A company should use the depreciation method, subject to IRS regulations, which gives it the maximum present value of the tax benefits of depreciation.

Let us find the after-tax cash flows for a project as follows:

Pre-tax income per year = E
Depreciation per year = D
Taxable income per year = $E - D$

Income tax rate = t
Income tax due = $t(E - D)$
Earnings after taxes = $E - t(E - D) = E - tE + tD = E(1 - t) + tD$

This gives us the after-tax cash flow as

$$C = E(1 - t) + tD \tag{4.3}$$

In the above equation, tD is called the *tax benefit of depreciation*. For a tax-exempt entity, such as a university, $t = 0$. In that case (4.3) reduces to $C = E$, meaning after-tax cash flow is the same s the pre-tax income. We should combine (4.1) and (4.3) to do the *NPV* calculations involving taxes and depreciation.

As an example, consider an asset with initial value \$50,000. The firm depreciates it with MACRS, with three-year life, as shown in the previous table. Assume that the firm uses 11% as the discount rate and its tax rate is 32%. Then the present value of tax benefits of depreciation is

$$= \frac{50{,}000(.3333)(.32)}{1.11} + \frac{50{,}000(.4444)(.32)}{1.11^2} + \frac{50{,}000(.1482)(.32)}{1.11^3} + \frac{50{,}000(.0741)(.32)}{1.11^4}$$

= \$13,090.08

In some cases, we have to include maintenance cost, or running cost, of a piece of equipment. Maintenance expense is a tax-deductible item and its net cost is $(1 - t)M$. The after tax cash flow in this situation is

$$C = E(1 - t) + tD - (1 - t)M$$

Or,

$$C = (1 - t)(E - M) + tD \tag{4.4}$$

4.4 Resale Value

A corporation may buy an asset, such as a car, or a machine, or a building, use it for a number of years, and then sell it. We should, therefore, consider the additional factor, the *resale value* of the machine. While using the asset, the corporation may depreciate the asset and get the corresponding tax benefit, tD. When a company sells a piece of equipment, it may, or may not, pay taxes on the sales price. It all depends upon the *book value* of the equipment. The book value, B, of a capital asset is the original value of the asset minus the depreciation already taken. For instance, if the initial value of a car is \$20,000 and it is depreciated at the rate of \$5000 per year, then its book value after one year is \$15,000, after two years \$10,000, after three years \$5,000, and after four years, when the car is fully depreciated, the book value is zero. By definition,

$$B = I_0 - nD \tag{4.5}$$

where I_0 is the price of the new equipment, n is the number of years it has been in service, and D is the (uniform) annual depreciation. For non-uniform depreciation, nD represents the total amount of depreciation. The book value of a brand new asset is I_0, whereas the book value of a fully depreciated asset is zero. Of course, the book value of an asset can never be negative.

The tax, T due at the time of selling a piece of equipment is the income tax rate, t multiplied by the difference between the sales price and the book value. Thus

$$T = t(S - B) \qquad (4.6)$$

where T = tax due, t = income tax rate, S = sale price of the equipment, and B = book value of the equipment. If T is negative, the company gets a tax credit. This will happen when the sale price is less than the book value of the asset. The after-tax value of the sales price S becomes W, where

$$W = S - T = S - t(S - B)$$

Put $B = I_0 - nD$ from (4.5)

$$W = S - t[S - (I_0 - nD)]$$

$$W = S - t(S - I_0 + nD) \qquad (4.7)$$

Use equation (4.7) to find the after-tax resale value of an asset.

The following table gives the corporate income tax rate in USA in 2007.

Taxable income over	Not over	Tax rate
$0	$50,000	15%
50,000	75,000	25%
75,000	100,000	34%
100,000	335,000	39%
335,000	10,000,000	34%
10,000,000	15,000,000	35%
15,000,000	18,333,333	38%
18,333,333	....	35%

Examples

4.8. Dora Corporation is planning to buy a machine for $10,000, which will result in $3,000 annual saving for the next five years. Dora will depreciate the machine in 5 years using the straight-line method and then sell it for $1500. The tax rate of Dora is 30%, and the proper discount rate is 15%. Should Dora make the investment?

A savings of $3,000 is equivalent to a pre-tax additional income of $3,000. The depreciation is 10,000/5 = $2,000 annually. The machine is fully depreciated after five years and the company pays taxes on resale value. After taxes, it is 1500(1 − .3) = \$1050. Using (4.3), we get the net cash flow as

$$C = 3000(1 - .3) + .3(2000) = \$2700$$

$$NPV = -10{,}000 + \sum_{i=1}^{5} \frac{2700}{1.15^i} + \frac{1050}{1.15^5} = -10{,}000 + \frac{2700(1 - 1.15^{-5})}{.15} + \frac{1050}{1.15^5} = -\$427.15$$

Based on the above considerations, Dora should reject the proposal. ♥

WRA -10000+sum((3000*(1-.3)+.3*10000/5)/1.15^i,i=1..5)+1500*(1-.3)/1.15^5

4.9. Tyree Corporation is considering the purchase of a machine, which will cost \$80,000. Tyree will depreciate it uniformly over 4 years although it will run for 5 years. It will then sell the machine for \$10,000. The tax rate of Tyree is 25%, and the proper discount rate is 11%. Find the minimum earnings before taxes generated by this machine that will make it profitable.

Suppose the minimum earnings before taxes is x. After taxes, it becomes

$$C = x(1 - t) + tD = x(1 - .25) + .25(80{,}000/4) = .75x + 5000$$

The tax benefit of depreciation, tD = \$5000, will continue for 4 years, while the other factor, .75x, will go on for 5 years. The company pays taxes on the resale value. After taxes, it becomes 10,000(1 − .25) = \$7500. Discounting all the cash flows and setting NPV = 0, we get

$$NPV = -80{,}000 + \sum_{i=1}^{5} \frac{.75x}{1.11^i} + \sum_{i=1}^{4} \frac{5000}{1.11^i} + \frac{7500}{1.11^5} = 0$$

Or, $$\sum_{i=1}^{5} \frac{.75x}{1.11^i} = 80{,}000 - \sum_{i=1}^{4} \frac{5000}{1.11^i} - \frac{7500}{1.11^5}$$

Or, $$2.771922763x = 60{,}036.88659$$

Or, $$x = \$21{,}658.93$$ ♥

The machine must generate at least \$21,658.93, before taxes, annually to be profitable.

WRA -80000+sum(.75*x/1.11^i,i=1..5)+sum(5000/1.11^i,i=1..4)+7500/1.11^5=0

4.10. Tompkins Farms needs a harvesting machine that will need \$4,000 in annual maintenance costs. Tompkins will depreciate the machine fully over 10 years and then sell it for 15% of its purchase price. It will save \$18,000 in labor costs annually. The tax rate of Tompkins is 30%, and the proper discount rate is 12%. How much should Tompkins pay for the machine just to break even?

Suppose the initial investment in the machine is I_0, which will result in the NPV to become zero. The tax benefit of depreciation per year, $tD = .3(I_0/10) = .03\ I_0$.

The annual cash flow C, including savings, maintenance expenses, depreciation and taxes, is given by (4.4) as

$$C = (1 - .3)(18{,}000 - 4{,}000) + .03\ I_0 = 9800 + .03\ I_0$$

The resale value is $.15I_0$. After taxes, it becomes $.15(1 - .3)\ I_0 = .105\ I_0$. Using a discount rate of 12%, (4.1) gives the NPV as

$$NPV = -I_0 + \sum_{i=1}^{10} \frac{9800 + .03\ I_0}{1.12^i} + \frac{.105\ I_0}{1.12^{10}} = 0$$

Isolating I_0,

$$I_0 \left[-1 + \sum_{i=1}^{10} \frac{.03}{1.12^i} + \frac{.105}{1.12^{10}} \right] = -\sum_{i=1}^{10} \frac{9800}{1.12^i}$$

Or,

$$-.7966861193\ I_0 = -55{,}372.18568$$

Or,

$$I_0 = \$69{,}503.14\ ♥$$

Tompkins should not buy the machine for more than $69,503.14. ♥

WRA -x+sum((9800+.03*x)/1.12^i,i=1..10)+.105*x/1.12^10=0

4.5 Hurdle Rate

As the course progresses, you will develop a better understanding of the concepts. Take for instance, the discount rate that a company uses in evaluating its projects. This depends on two factors. The first factor is the cost of capital of a firm, which we will discuss in Chapter 9. If the cost of capital for a firm is 12%, then the minimum acceptable rate of return, and the discount rate, for a given project must be 12%.

The second factor is risk. The measurement of risk is more difficult. We will talk about it in Chapter 6 and 7. Suppose a firm, with cost of capital 12%, is undertaking a rather risky project. The risk-adjusted discount rate will be, perhaps, 14 or 15%. Additional source of risk is due to leveraging, which will further increase the cost of capital. We will discuss leveraged beta of a firm in chapter 11. Calculating the discount rate is a little complicated but after completing this course, I hope you will have a better feel for this number.

In real life, firms use a "hurdle rate," an arbitrarily chosen discount rate, which tends to be too high, perhaps 25% or 30%. Otherwise, they will calculate the internal rate of return of a project. If it is more than say 25%, they will accept the project. By using a high discount rate, they are just playing it safe, but they are also rejecting many reasonable projects with positive NPV.

In Chapter 15, we will look at the analysis of investment opportunities. We have to look at the proper discount rate again.

The US government occasionally provides investment tax credit to corporations to encourage investment in plant and equipment. This gives a boost to the economic development. For example, if 10% tax credit is available, the initial investment is 90% of the price of the equipment and the remaining cost is deductible from the income tax of the company. The total depreciation depends on the actual cost of the equipment, which is, in this case 90% of the price of the equipment. Currently, however, this tax credit is not available.

4.11. Grissom Corporation is planning to buy a new computer and is considering three alternatives. The price of each computer, along with its annual revenues and maintenance expenses, assumed to be at the end of each year, are shown below.

Brand	Price	Revenue	Maintenance
IBM	$80,000	$20,000	$4,000
Dell	70,000	18,000	3,000
Gateway	100,000	25,000	5,000

An investment tax credit of 10% is available and the company will depreciate the computers over a 10-year period with no residual value. Grissom is in the 40% marginal tax bracket and its hurdle rate is 15%. Which computer, if any, should Grissom acquire?

Consider IBM computer first. Its list price is $80,000, so Grissom is able to acquire it for .9(80,000) = $72,000. Grissom will depreciate it over 10 years, thus the annual depreciation is $7200.

Maintenance expense is a tax-deductible item and its after-tax cost is $(1 - t)M$. The after-tax cash flow in this problem is

$$C = (1 - t)(E - M) + tD \tag{4.4}$$

For IBM, $C = (1 - .4)(20{,}000 - 4{,}000) + .4(72{,}000/10) = \$12{,}480$

$$NPV = -72{,}000 + \sum_{i=1}^{10} \frac{12{,}480}{1.15^i} = -72{,}000 + \frac{12{,}480(1 - 1.15^{-10})}{.15} = -\$9366$$

For Dell, $C = (1 - .4)(18{,}000 - 3{,}000) + .4(63{,}000/10) = \$11{,}520$

$$NPV = -.9(70{,}000) + \sum_{i=1}^{10} \frac{11{,}520}{1.15^i} = -.9(70{,}000) + \frac{11{,}520(1 - 1.15^{-10})}{.15} = -\$5184$$

For Gateway, $C = (1 - .4)(25{,}000 - 5{,}000) + .4(90{,}000/10) = \$15{,}600$

$$NPV = -.9(100{,}000) + \sum_{i=1}^{10} \frac{15{,}600}{1.15^i} = -.9(100{,}000) + \frac{15{,}600(1 - 1.15^{-10})}{.15} = -\$11{,}707$$

Because all computers have a negative *NPV*, the firm should reject them all. However, relative to one another, Dell computer is the best investment. ♥

4.12. The Cameron Company has these three options to buy a new machine:

Machine	Cost	Maintenance	Cost of capital
Alpha	$40,000	$5000	10%
Beta	$45,000	$4500	9%
Gamma	$50,000	$4000	8%

All the machines are identical in nature and each one will generate pre-tax revenue of $18,000 annually. The company will depreciate any of the machines on a straight-line basis over next 5 years with no residual value. The company is in 35% tax bracket. Which machine should Cameron buy?

It is possible to have different discount rates. For instance, the manufacturer of the machine may offer to loan the money to the firm to buy the machine at a lower rate to compensate for the higher price of the equipment. The annual cash flow for each machine and its NPV are as follows:

Alpha, $C = (1 - .35)(18{,}000 - 5{,}000) + .35(8{,}000) = \$11{,}250$

$$NPV = -40{,}000 + \sum_{i=1}^{5} \frac{11{,}250}{1.1^i} = -40{,}000 + \frac{11{,}250(1 - 1.1^{-5})}{.1} = \$2646$$

Beta, $C = (1 - .35)(18{,}000 - 4{,}500) + .35(9{,}000) = \$11{,}925$

$$NPV = -45{,}000 + \sum_{i=1}^{5} \frac{11925}{1.09^i} = -45{,}000 + \frac{11925(1 - 1.09^{-5})}{.09} = \$1384$$

Gamma, $C = (1 - .35)(18{,}000 - 4{,}000) + .35(10{,}000) = \$12{,}600$

$$NPV = -50{,}000 + \sum_{i=1}^{5} \frac{12{,}600}{1.08^i} = -50{,}000 + \frac{12{,}600(1 - 1.08^{-5})}{.08} = \$308$$

Machine Alpha is the best buy on this basis. ♥

4.13. The Adams Company is planning to buy a new computer for $80,000 that will increase the pre-tax earnings of the company by $30,000 annually. The company will depreciate the computer fully on a straight-line basis over a 5-year period, and then sell it for $10,000. The company has a tax rate of 40%. If the after-tax cost of capital of Adams is 11%, should it purchase the computer?

This is a problem involving the resale value of the equipment. When the company sells the fully depreciated computer, its book value is zero. The tax applicable on the sale is thus tS, by equation (4.4). The after-tax value of the sale is $(1 – t)S = (1 – .4)(10{,}000) =$ \$6,000. This money is available after 5 years, and we must find its present value.

The cash flow is $C = (1 – .4)(30{,}000) + .4(16{,}000) = \$24{,}400$

Including the sale of the equipment,

$$\text{NPV} = -80{,}000 + \sum_{i=1}^{5} \frac{24{,}400}{1.11^i} + \frac{6000}{1.11^5} \qquad \text{(A)}$$

This gives NPV = \$13,741. It is a very good investment. Buy it. ♥

To solve the problem at WolframAlpha, write equation (A) as

WRA `-80000+sum(24400/1.11^i,i=1..5)+6000/1.11^5`

4.14. Carver Corporation is planning to buy a machine costing \$50,000, and it will depreciate it fully along a straight line over 5 years. The machine will generate unknown earnings before interest and taxes (*EBIT*), which will remain constant for the first 5 years and then drop to half that value during the next five years. The tax rate of Carver is 30%, and its discount rate is 10%. Calculate the *EBIT* for the machine to just break even, that is, have zero *NPV*.

Suppose the unknown *EBIT* is E. The tax benefits of depreciation are available only during the first five years. Then the cash flows are:

First five years, $C = E(1 – .3) + .3(10{,}000) = .7\,E + 3000$
Next five years, $C = (E/2)(1 – .3) = .35\,E$

To break even, set NPV = 0,

$$\text{NPV} = -50{,}000 + \sum_{i=1}^{5} \frac{.7\,E + 3000}{1.1^i} + \sum_{i=6}^{10} \frac{.35\,E}{1.1^i} = 0 \qquad \text{(A)}$$

Isolating the terms containing E, we get

$$-50{,}000 + \sum_{i=1}^{5} \frac{3000}{1.1^i} + \sum_{i=1}^{5} \frac{.7\,E}{1.1^i} + \sum_{i=6}^{10} \frac{.35\,E}{1.1^i} = 0$$

Put $$\sum_{i=6}^{10} \frac{.35\,E}{1.1^i} = \frac{1}{1.1^5} \sum_{i=1}^{5} \frac{.35\,E}{1.1^i}$$ in the above equation,

$$-50{,}000 + \frac{3000(1-1.1^{-5})}{0.1} = -E\left(\frac{0.7(1-1.1^{-5})}{0.1} + \frac{1}{1.1^5}\,\frac{0.35(1-1.1^{-5})}{0.1}\right)$$

Or, $$38{,}627.64 = 3.4773739\,E$$

Or, $$E = \$11{,}108$$ ♥

The machine must generate \$11,108 per year for the first five years, and then half that amount for the next five years just to break even.

To solve the problem using WolframAlpha, write equation (A) as

WRA -50000+sum((.7*x+3000)/1.1^i,i=1..5)+sum(.35*x/1.1^i,i=6..10)=0

4.15. Gray Metals Company needs a new machine, which would save the company \$3,000 annually for the first five years and then \$2,000 annually for another five years. Gray will depreciate the machine on a straight-line basis for 10 years. Gray is in the 40% tax bracket and its after-tax cost of capital is 8%. What is the break-even price of the machine for Gray?

Suppose the break-even price is x, which should equal the discounted future cash flows, including the tax benefits of depreciation. If the price of the machine is x, then depreciation per year is $x/10$ or $0.1x$ and the corresponding tax advantage is $0.1(x)(0.4) = 0.04x$. The quantity $E(1 - t)$ for the first five years is $3000(1 - .4) =$ \$1800, and for the next five years, it is $2000(1 - .4) =$ \$1200.

Consider the present value of three sets of cash flows:
(1) PV of $E(1 - t) =$ \$1800 annually for years 1-5
(2) PV of $E(1 - t) =$ \$1200 annually for years 6-10
(3) PV of tax benefits of depreciation, $tD = 0.04x$ for years 1-10

The sum of these is the price of the machine x. Thus

$$x = \sum_{i=1}^{5} \frac{1800}{1.08^i} + \sum_{i=6}^{10} \frac{1200}{1.08^i} + \sum_{i=1}^{10} \frac{.04x}{1.08^i} \qquad \text{(A)}$$

Or,
$$x\left(1 - \sum_{i=1}^{10} \frac{.04}{1.08^i}\right) = \sum_{i=1}^{5} \frac{1800}{1.08^i} + \frac{1}{1.08^5} \sum_{i=1}^{5} \frac{1200}{1.08^i}$$

Or
$$x\left(1 - \frac{0.04(1 - 1.08^{-10})}{0.08}\right) = \frac{1800\,(1 - 1.08^{-5})}{0.08} + \frac{1}{1.08^5}\,\frac{1200\,(1 - 1.08^{-5})}{0.08}$$

Or,
$$x(.7315967) = 10{,}447.72$$

Or,
$$x = 14{,}280.71$$

The break-even price of the machine is \$14,280.71. ♥

To solve the problem using WolframAlpha, write equation (A) as

WRA x=sum(1800/1.08^i,i=1..5)+sum(1200/1.08^i,i=6..10)+sum(.04*x/1.08^i,i=1..10)

4.16. Monroe Corporation needs a machine, which will cost \$100,000. Monroe will depreciate it on a straight-line basis over 5 years with no resale value. The tax rate of Monroe is 28%, and its after-tax cost of capital is 11%. The machine will have an *EBIT* of \$18,000 a year for the first five years, and then an uncertain amount for the next five years, years 6 through 10. Find the minimum amount of this uncertain *EBIT*, which will make the purchase of this machine acceptable.

Suppose the unknown *EBIT* for the years 6 through 10 is E. The cash flows are:

For first five years, $C = 18{,}000(1 - .28) + .28(20{,}000) = \$18{,}560$
For next five years, $C = E(1 - .28) = .72\,E$

Setting *NPV* equal to zero, we have

$$NPV = -100{,}000 + \sum_{i=1}^{5} \frac{18{,}560}{1.11^i} + \sum_{i=6}^{10} \frac{.72\,E}{1.11^i} = 0 \qquad \text{(A)}$$

Or, $$100{,}000 - \frac{18{,}560(1 - 1.11^{-5})}{0.11} = \frac{1}{1.11^5} \sum_{i=1}^{5} \frac{.72\,E}{1.11^i} = \frac{1}{1.11^5}\left(\frac{0.72E(1 - 1.11^{-5})}{0.11}\right)$$

Or, $$E = \left[100{,}000 - \frac{18{,}560(1 - 1.11^{-5})}{0.11}\right](1.11^5)\left(\frac{.11}{0.72(1 - 1.11^{-5})}\right)$$

which gives $E = \$19{,}886$ ♥

To solve the problem using WolframAlpha, write equation (A) as

WRA `-100000+sum(18560/1.11^i,i=1..5)+sum(.72*x/1.11^i,i=6..10)=0`

4.6 Projects with Unequal Lives

All the problems that we have discussed so far involve either a single project, or two projects with equal lives. In real life, we may have to compare the performance of two machines with different lives. One machine may be cheaper, but it will not last as long as a more expensive one. How can we possibly compare two machines with different lives?

One way to handle a problem like this one is to assume that you will continue to replace a machine by a similar one forever. This means that the life of the project is infinity in each case, but this life is spanned by one kind of machine or the other. Consider the following problem.

4.17. You want to install a new heat pump in your house. Two different models are available, Shinn and Gardner. They are both satisfactory in performance. Their characteristics are as follows:

Heat Pump	Cost	Life	Annual expenses
Shinn	$3000	4 years	$700
Gardner	$4000	5 years	$600

The proper discount rate is 12%. Which unit should you buy?

Assume that you cannot depreciate the equipment in your personal home. The problem requires careful analysis since the machines have different lives. Assume that you will replace the machines every 4 or 5 years forever. For the first unit, the *NPV* is

$$NPV = -3000 - \frac{700}{1.12} - \frac{700}{1.12^2} - \frac{700}{1.12^3} - \frac{700}{1.12^4} - \frac{3000}{1.12^4} - \frac{700}{1.12^5} - \frac{700}{1.12^6} - \ldots \qquad \text{(A)}$$

The terms in the blue color represent the cost of new equipment and its replacement after four years. The remaining terms are due to the annual expense of the heat pump. The negative signs mean cash outflows. The entire series is consisting of two infinite series. The ratio of the blue terms with numerator 3,000 is $\frac{1}{1.12^4}$, and the ratio for the other terms with numerator 700 is 1/1.12. Using (1.5), we get

$$NPV = -\frac{3000}{1 - \frac{1}{1.12^4}} - \frac{700}{1.12\,(1 - 1/1.12)} = -\$14{,}064.19$$

This figure, \$14,064.19, represents the total cost (measured in present dollars) of using the Shinn heat pump for an infinitely long period. The calculations for the Gardner unit are done similarly. Change the cost of the equipment from \$3000 to \$4000; life of the machine from 4 to 5 years, and annual cost from \$700 to \$600. Making these changes, for the Gardner unit, we get

$$NPV = -\frac{4000}{1 - \frac{1}{1.12^5}} - \frac{600}{1.12\,(1 - 1/1.12)} = -\$14{,}246.99$$

Comparing the total costs, it is better to buy the Shinn Heat unit. ♥

To do the calculation using WolframAlpha, write equation (A) as

WRA `-sum(3000/1.12^(4*i),i=0..infinity)-sum(700/1.12^i,i=1..infinity)`

WRA `-sum(4000/1.12^(5*i),i=0..infinity)-sum(600/1.12^i,i=1..infinity)`

A second approach to do these problems is to find the *equivalent annual cost*. This cost is the sum of the payments that will amortize the purchase price of the equipment, plus the cost of running it. First, we find the amount needed to amortize the purchase cost of the heat-pump. For Shinn, the amortization amount *A* is given by

$$3000 = \sum_{i=1}^{4} \frac{A}{1.12^i}$$

Solving it, we get A = \$987.70. This means that either we can pay \$3000 to buy the unit now, or we may pay \$987.70 every year for the next four years. Next, we add the annual running cost, \$700, to it. Then we get the total *equivalent annual cost* to be 987.70 + 700 = \$1687.70.

Following the same procedure for the other unit, we get

$$4000 = \sum_{i=1}^{5} \frac{A}{1.12^i}$$

This gives A = \$1109.64. The total equivalent annual cost is thus 1109.64 + 600 = \$1709.64. Comparing the two costs, we find that Shinn unit is cheaper.

If we find the PV of these costs in perpetuity, it comes out to be 1687.70/.12 = \$14,064.17 for Shinn, and 1709.64/.12 = \$14,247.00 for Gardner. These numbers are the same as found by the previous method. Thus, the two methods are equivalent.

4.18. Djibouti Corporation is considering the purchase of an air conditioning unit and it has these two choices.

Unit	Initial cost	Annual cost	Expected life
A	\$80,000	\$10,000	5 years
B	\$70,000	\$8,000	4 years

Djibouti is in the 40% tax bracket. Which unit should it buy? Assume that it will depreciate each piece of equipment on a straight-line basis with no residual value during its working life. The proper discount rate is 10%.

In this problem, we are concerned with three items: (1) The replacement cost of the unit every four or five years (2) the annual after-tax cost of electricity and (3) the annual tax benefits of depreciation of the unit.

For Unit A, the replacement cost is \$80,000 every five years, the after-tax cost of electricity is (1 – .4)(10,000) = \$6,000 annually, and the tax benefit from depreciation is .4(80,000/5) = \$6400. Combining the after-tax cost of electricity and the tax benefit of depreciation, we have a net benefit of 6400 − 6000 = \$400 annually. The NPV for an infinite period is thus

$$NPV(\text{A}) = -\,80{,}000 - \frac{80{,}000}{1.1^5} - \frac{80{,}000}{1.1^{10}} - \ldots + \frac{400}{1.1} + \frac{400}{1.1^2} + \frac{400}{1.1^3} + \ldots \infty$$

$$= -\frac{80{,}000}{1 - 1/1.1^5} + \frac{400}{0.1} = -\$207{,}038$$

For the second unit, the replacement cost is \$70,000 every four years, the annual after-tax cost of electricity is .6(8000) = \$4800, and the tax benefit of depreciation is .4(70,000/4) = \$7000. Combining the last two we have a benefit of 7000 – 4800 = \$2200. Using the previous calculation as a guide, we have

$$NPV(\text{B}) = -\frac{70\ 000}{1 - 1/1.1^4} + \frac{2200}{0.1} = -\$198{,}830$$

Based on these calculations, unit B is somewhat cheaper. ♥

Let us solve this problem using *equivalent annual cost*. The equivalent annual cost for replacing the first unit is found from

$$80{,}000 = \sum_{i=1}^{5} \frac{A}{1.1^i}$$

This gives us $$80{,}000 = \frac{A(1 - 1.1^{-5})}{.1}$$

Or, $$A = \frac{80\ 000{*}.1}{1 - 1.1^{-5}} = \$21{,}103.80$$

The tax benefit due to depreciation per year is .4*80,000/5 = \$6400. The after-tax cost of electricity is (1 – .4)10,000 = \$6000. Thus the total cost for Unit A is 21,103.80 – 6400 + 6000 = \$20,703.80.

The replacement cost for Unit B is found as $B = \frac{70{,}000{*}.1}{1 - 1.1^{-4}}$ = \$22,082.96. The tax benefit due to depreciation per year is .4*70,000/4 = \$7000. The after-tax cost of electricity is (1 – .4)8,000 = \$4800. Thus the total cost for Unit B is 22,082.96 – 7000 + 4800 = \$19,882.96.

Comparing their costs, \$20,703.80 and \$19,882.96, Unit B is cheaper,

To reconcile the answer, we also calculate the total cost for an infinite time horizon, we get 20,703.80/.1 = \$207,038 and 19,882.96/.1 = \$198,830, as before. ♥

Problems

4.19. The Scranton Times is planning to buy a new press for \$120,000 that will save the company \$30,000 annually. The press has a useful life of 10 years. The Times has a tax rate of 40%, and it will depreciate the machine on a straight-line basis. The after-tax cost of capital of the Times is 9.6%. Should it buy the new press? *NPV* = \$22,536.20, yes ♥

4.20. Ellsmere Corporation plans to buy a new machine for $50,000, which will save the company $12,000 annually. Ellsmere will depreciate the machine on the ACRS with three-year life, the annual depreciation being 29%, 47%, and 24%. The company expects that the machine will run for 5 years, and then it will sell it for $5,000. The after-tax cost of capital to the company is 8%, and its tax rate is 40%. Should Ellsmere buy the machine? *NPV* = – $1,970.99, no ♥

4.21. Cline Incorporated wants to buy a machine for $24,000, and depreciate it on straight-line basis over 6 years. Cline has marginal tax rate of 35% and its after-tax cost of capital is 7%. Calculate the minimum pre-tax annual earnings generated by this machine to justify its purchase. $5592.46 ♥

4.22. Allen Corporation has to decide between the following two air conditioning units for an office building. Both units are adequate in their performance.

	Carrier	**Worthington**
Initial cost	$120,000	$80,000
Annual maintenance cost	$10,000	$12,000
Annual electricity cost	$20,000	$25,000
Expected life	6 years	5 years

The company will use straight-line depreciation, with no resale value. The tax rate of Allen is 28%, and the proper discount rate is 10%. Which one of these units will prove to be less costly in the long run?

NPV(Carrier) = –$435,529, NPV(Worthington) = –$432,638 (cheaper) ♥

4.23. Alcott Corp is interested in buying a machine for $40,000. It will depreciate the machine uniformly to zero value over a 5-year period. During this period, the machine will add $8,000 annually to the *EBDIT* of the company. Finally, Alcott will sell the machine for $5,000. The tax rate of Alcott is 40% and the proper discount rate is 9%. Should Alcott buy the machine? *NPV* = –$6,933, no ♥

4.24. Darwin Corporation is going to buy a machine for $152,000 that will save the company $20,000 annually. Darwin will depreciate the machine completely in five years using straight-line method. The tax rate of company is 30%, and it uses a discount rate of 12%. Show that this machine will never be profitable.

Multiple Choice Questions

1. The discount rate of a project does not depend upon the

A. risk of project C. prevailing interest rates
B. cost of capital of firm D. life of project

2. The after-tax cash flow of a project does not depend upon the

A. income tax rate of company C. riskless interest rate
B. pre-tax earnings from project D. annual depreciation of assets

3. The tax due at the time of the sale of an asset does not depend upon the

A. income tax rate of firm C. sale value of asset
B. book value of asset D. discount rate for project

4. Define: T = tax due at the time of sale of an asset, t = income tax rate, B = book value of the asset, S = its sale price. The relationship between these quantities is

A. $T = (B - S)(1 - t)$ C. $T = (B - S)t$
B. $T = (S - B)(1 - t)$ D. $T = (S - B)t$

Key Terms