

1. FINANCE AND ANALYTICAL TOOLS

Objectives: After reading this chapter, you will be able to

1. Get an overview of finance and basic algebra.
2. Use geometric series in financial calculations.
3. Understand the basic concepts of statistics.
4. Use Excel, Maple, or [WolframAlpha](#) to solve mathematical problems.

1.1 Field of Finance

When you look at the [balance sheet](#) of a company, you will see the assets and liabilities are categorized as long-term, or short-term. In this course, we are dealing with the management of long-term assets (machinery, equipment, buildings, land, etc.) and long-term liabilities ([bonds](#)) and other long-term financing ([stocks](#), [preferred stock](#)). Another course, [FIN 361](#), Working Capital Management, deals with the management of current assets (cash, [marketable securities](#), [accounts receivable](#), and [inventories](#)) and current liabilities ([accounts payable](#), short-term financing, and [accruals](#)).

The following diagram outlines the relationship between the short-term and the long-term assets and liabilities.

	Assets	Liabilities and Equity	
FIN 361	Current Assets Cash Marketable Securities Accounts Receivable Inventories	Current Liabilities Accounts Payable Accruals Notes Payable	Short term
FIN 508	Long-term Assets Plant and Equipment Less Accumulated Depreciation = Net Plant and Equipment	Long-term Liabilities Long-term bonds Owners' Equity Common Stock Preferred Stock Retained Earnings	Long term
	Total Assets are equal to	Total Liabilities and Equity	

The above diagram represents the balance sheet of a company. It is the snapshot of the financial condition of a corporation on a certain date. The last line in the above table shows a very important concept in finance,

Total Assets are equal to Total Liabilities and Equity

In symbols, one can write it as

$$V = B + S \quad (1.1)$$

In this equation,

V = total market value of the company

B = total market value of the liabilities of the company. This includes bonds and short-term notes.

S = total market value of the stock of the company. This includes common and preferred stock.

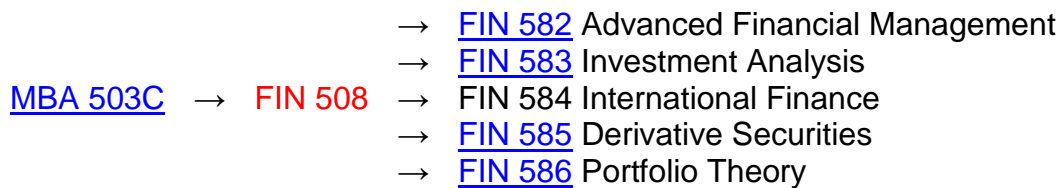
We will discuss this equation in more detail in chapter 9.

First, one should learn the basic financial principles, such as time value of money, risk, options, cash flows, and the measurable quantities such as the stock price, earnings per share, debt ratio, etc. FIN 508, explains these basic ideas, and their application to long-term financial management of a company.

You may say, “If I were a CFO of a company, I would monitor the spending daily.” You will be focusing on the **tan** part of the diagram, which is a valid activity. You will monitor cash, accounts receivables, inventory, and the daily cash flows. However, you have to know the whole picture. The most important decisions are made in the **blue** part of the diagram. Here you are raising and investing millions, perhaps billions, of dollars. The recent IPO of [Facebook](#) (May 18, 2012) is a good example of this activity. If you make an error, it may turn out to be a big mistake.

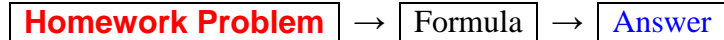
When you are making a financial decision at a company, you cannot rely entirely on gut feeling, just because you know your business. You have to do some homework first and find the right course of action. If you are presenting your case to a board of directors, or in a court of law, you have to convince the audience with facts. We are learning to do that here in this course.

The relationship between different finance courses is as follows.



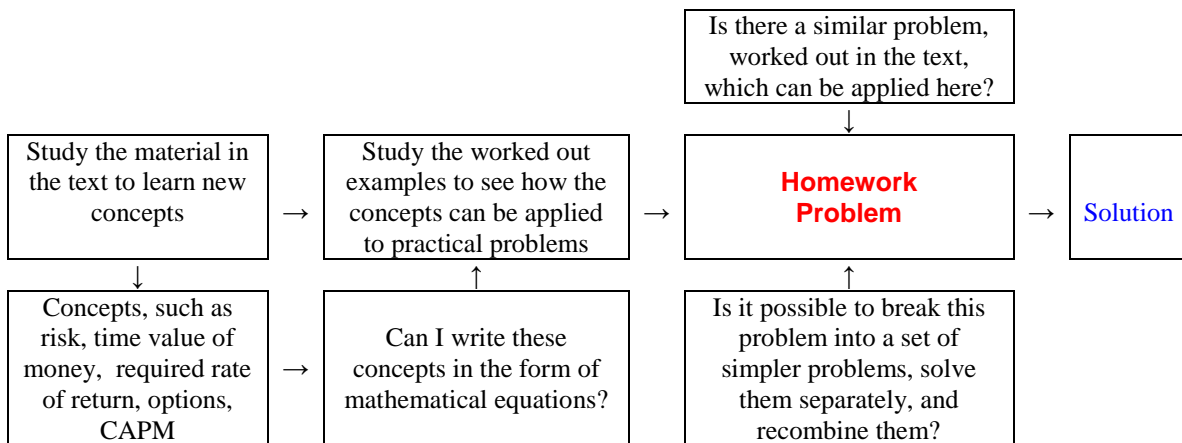
1.2 Problem Solving

One can learn finance efficiently by learning to solve financial problems analytically. This textbook has plenty of problems, many of them are solved examples and the others are the end-of-the-chapter exercises. There are two ways to look at any homework problem. The first one is the quick one:



Some students have the temptation to solve the problem quickly without understanding the concept that the problem is supposed to develop. They miss the real purpose of the exercise, which is to consolidate an idea and observe its application.

There is another way to look at the same problem.



The second method is obviously more cumbersome, but it helps the student understand the material.

Before we actually start studying finance and the financial management as a discipline, it is worthwhile to review some of the fundamental concepts in mathematics first. This will help us appreciate the usefulness of analytical techniques as powerful tools in financial decision-making. We shall briefly review elementary algebra, basic concepts in statistics, and finally learn Excel, Maple, or [WolframAlpha](#) as a handy way to cut through the mathematical details.

Our approach toward learning finance is to translate a word problem into a mathematical equation involving some unknown quantity, solve the equation, and get the answer. This will help us determine an exact answer, rather than just an approximation. This will lead to a better decision.

1.3 [Video 01A](#), Linear Equations

To review the basic concepts of algebra, we look at the simplest equations first, the [linear equations](#). These equations do not have any squares, square roots, or trigonometric or other complicated mathematical functions.

Example

1.0. Suppose John buys 300 shares of AT&T stock at \$26 a share and pays a commission of \$10. When he sells the stock, he will have to pay \$10 in commission again. Find the selling price of the stock, so that after paying all transaction costs, John's profit is \$200.

Let us define profit π as the difference between the final payoff F , after commissions, and the initial investment I_0 , including commissions. We can write it as a linear equation as follows

$$\pi = F - I_0$$

We require a profit of \$200, thus, $\pi = 200$. Suppose the final selling price of the stock per share is x , the number we want to calculate. The initial investment in the stock, including commission, is $I_0 = 300(26) + 10 = \$7810$. Selling 300 shares at x dollars each, and paying a commission of \$10, gives the final payoff as, $F = 300x - 10$. Make these substitutions in the above equation to obtain

$$200 = 300x - 10 - 7810$$

Moving things around, we get $200 + 10 + 7810 = 300x$

Or,
$$8020 = 300x$$

Or,
$$x = 8020/300 = 26.73333333 \approx \$26.73 \heartsuit$$

This means that the stock price should rise to \$26.73 to get the desired profit. Note that the answer has a dollar sign and it is truncated to a reasonable degree of accuracy, namely, to the nearest penny.

Next, consider a somewhat more complicated problem involving dollars, doughnuts, and coffee.

1.1. Jane works in a coffee shop. During the first half-hour, she sold 12 cups of coffee and 6 doughnuts, and collected \$33 in sales. In the next hour, she served 17 cups of coffee and sold 8 doughnuts, for which she received \$46. Find the price of a cup of coffee and that of a doughnut.

This is an example where we have to find the value of two unknown quantities. The general rule is that you need two equations to find two unknowns. We have to develop two equations by looking at the sales in the first half-hour and in the second hour. Suppose the price of a cup of coffee is x dollars, and that of a doughnut y dollars.

First half-hour, 12 cups and 6 doughnuts for \$33, gives	$12x + 6y = 33$
Second hour, 17 cups and 8 doughnuts for \$46, gives	$17x + 8y = 46$

Now we have to solve the above equations for x and y .

First, try to eliminate one of the variables, say y . You can do this by multiplying the first equation by 8 and the second one by 6, and then subtracting the second equation from the first. This gives

$$\begin{aligned} 8 \cdot 12x + 8 \cdot 6y &= 8 \cdot 33 \\ 6 \cdot 17x + 6 \cdot 8y &= 6 \cdot 46 \end{aligned}$$

Subtracting second from first, $(8*12 - 6*17) x = 8*33 - 6*46$

Simplifying it, $-6 x = -12$, or $x = 2$ ♥

Substituting this value of x in the first equation, we have $12*2 + 6 y = 33$

Or, $6y = 33 - 24 = 9$

$$y = 9/6 = 3/2 \text{ ♥}$$

The answer is that a cup of coffee sells for \$2 and a doughnut for \$1.50.

1.3 WolframAlpha

[Mathematica](#) is a useful analytical software, which has capabilities similar to Maple. It can perform all the mathematical problems equally well. [Mathematica](#) has a website at [WolframAlpha](#), which is free to use. The instructions at [WolframAlpha](#) are almost identical to those in Maple. You should explore this website and use it when you do not have access to Maple. In this text, you can access [WolframAlpha](#) by clicking on this button [WRA](#).

For instance, to solve the equations in example 1.1

$$\begin{aligned} 12x + 6y &= 33 \\ 17x + 8y &= 46 \end{aligned}$$

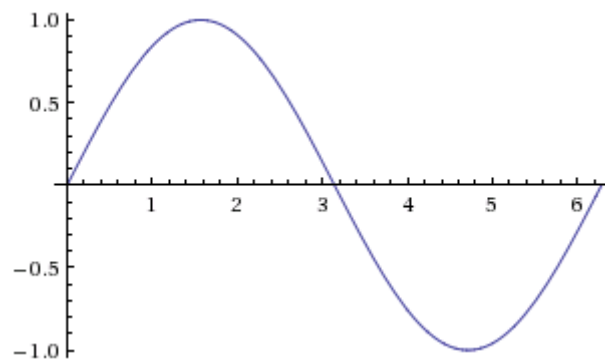
for x and y , enter the instructions as follows:

[WRA](#) `solve(12*x+6*y=33,17*x+8*y=46)`

It provides the solution as $x = 2$, $y = 3/2$.

To see the sine wave of Figure 1.1, write

[WRA](#) `plot(sin(x), x=0..2*Pi)`



1.2 Non-linear Equations

Non-linear equations contain higher powers of the unknown variable, or the variable itself may show up in the power of a number. For instance, a quadratic equation is a non-linear equation. The general form of a quadratic equation is

$$ax^2 + bx + c = 0 \quad (1.2)$$

The roots of this equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (1.3)$$

Consider the following examples of non-linear equations.

Examples

1.2. Solve for x : $1.113^x = 2.678$

First, we recall the basic property of logarithm functions, namely,

$$\ln(a^x) = x \ln a$$

Taking the logarithm on both sides of the given equation, we obtain

$$x \ln(1.113) = \ln(2.678)$$

Or,
$$x = \frac{\ln(2.678)}{\ln(1.113)} = \frac{0.9850702}{0.1070591} = 9.201 \heartsuit$$

You can save some time by doing the calculation at [WolframAlpha](#) as follows:

[WRA](#) $1.113^x = 2.678$

1.3. Solve for x , $(2 + x)^{2.11} = 16.55$

This gives
$$2 + x = (16.55)^{1/2.11}$$

Or,
$$x = (16.55)^{1/2.11} - 2 = 1.781 \heartsuit$$

You can verify the answer at [WolframAlpha](#) as follows:

[WRA](#) $(2+x)^{2.11}=16.55$

1.4. Find the roots of $5x^2 + 6x - 11 = 0$

This is a typical quadratic equation. Use equation (1.2) and put $a = 5$, $b = 6$, and $c = -11$. This gives

$$x = \frac{-6 \pm \sqrt{36 - 4(5)(-11)}}{10} = \frac{-6 \pm \sqrt{256}}{10} = \frac{-6 \pm 16}{10} = -\frac{11}{5} \text{ or } 1 \heartsuit$$

You can verify the answer at [WolframAlpha](#) as follows:

[WRA](#) `5*x^2+6*x-11=0`

1.3 Geometric Series

In many financial management problems, we have to deal with a series of cash flows. When we look at the present value, or the future value, of these cash flows, the resulting series is a geometric series. Thus, geometric series will play an important role in managing money. Let us consider a series of numbers represented by the following sequence

$$a, ax, ax^2, ax^3, \dots, ax^{n-1}$$

The sequence has the property that each number is multiplied by x to generate the next number in the list. There are altogether n terms in this series, the first one has no x , the second one has an x , and the third one has x^2 . By this reasoning, we know that the n th term must have x^{n-1} in it. This type of series is called a *geometric series*. Our concern is to find the sum of such a series having n terms with the general form

$$S = a + ax + ax^2 + ax^3 + \dots + ax^{n-1} \quad (1.4)$$

To evaluate the sum, proceed as follows. Multiply each term by x and write the terms on the right side of the equation one-step to the right of their original position. We can set up the original and the new series as follows:

$$\begin{aligned} S &= a + ax + ax^2 + ax^3 + \dots + ax^{n-1} \\ xS &= ax + ax^2 + ax^3 + \dots + ax^{n-1} + ax^n \end{aligned}$$

If we subtract the second equation from the first one, most of the terms will cancel out, and we get

$$S - xS = a - ax^n$$

Or,
$$S(1 - x) = a(1 - x^n)$$

or,
$$S_n = \frac{a(1 - x^n)}{1 - x} \quad (1.5)$$

This is the general expression for the summation of a geometric series with n terms, the first term being a , and the ratio between the terms being x . This is a useful formula, which we can use for the summation of an [annuity](#).

If the number of terms in an annuity is infinite, it becomes a [perpetuity](#). To find the sum of an infinite series, we note that when n approaches infinity, $x^n = 0$ for $x < 1$. Thus, the sum for an infinite geometric series becomes

$$S_{\infty} = \frac{a}{1-x} \quad (1.6)$$

To obtain equations (1.4) and (1.5) at [WolframAlpha](#), use the following instructions:

[WRA](#) `sum(a*x^i,i=0..n-1)`

[WRA](#) `sum(a*x^i,i=0..infinity)`

Examples

1.5. Find the sum of $3 + 6 + 12 + 24 \dots$, 13 terms

We identify $a = 3$, $x = 2$, and $n = 13$. Putting these in (1.4), we get

$$S = \frac{a(1-x^n)}{1-x} = \frac{3(1-2^{13})}{1-2} = 3(2^{13}-1) = 24,573 \heartsuit$$

To verify the answer at [WolframAlpha](#), use the following instruction:

[WRA](#) `sum(3*2^i,i=0..12)`

1.6. Find the sum of $1.7 + 2.21 + 2.873 \dots$, 11 terms

Here $a = 1.7$, and $x = 2.21/1.7 = 1.3$. Also, $n = 11$. This gives

$$S = \frac{a(1-x^n)}{1-x} = \frac{1.7(1-1.3^{11})}{1-1.3} = \frac{1.7(1.3^{11}-1)}{1.3-1} = 95.89 \heartsuit$$

At [WolframAlpha](#), use the following instruction.

[WRA](#) `sum(1.7*1.3^i,i=0..10)`

1.7. Find the value of $\sum_{i=1}^{100} \frac{25}{1.12^i}$

The mathematical expression possibly means a sum of one hundred annual payments of \$25 each, discounted at the rate of 12% per annum. Write it as

$$\sum_{i=1}^{100} \frac{25}{1.12^i} = \frac{25}{1.12} + \frac{25}{1.12^2} + \frac{25}{1.12^3} + \frac{25}{1.12^4} + \dots + \frac{25}{1.12^{100}}$$

The distribution in the previous example is a [discrete probability distribution](#). Another example of such a distribution is the set of probabilities for the outcomes of a roll of dice. With a single die, the probability is 1/6 each of getting a 1, or 2, or 3, and so on.

A probability distribution may be continuous, such as the [normal probability distribution](#). The probability distribution describing the life expectancy of human beings, or machines, is a continuous distribution. At present, we shall try to describe the uncertainty in terms of discrete probability. We are going to use a subjective probability distribution to describe the uncertain future.

We can find the expected value of a certain quantity by multiplying the probability of each outcome by the value of that outcome.

Example

1.8. A project has the following expected cash flows

State of the Economy	Probability	Cash Flow X
Good	60%	\$10,000
Fair	30%	\$6,000
Poor	10%	\$2,000

To find the expected cash flow, we compute

$$E(X) = .6(\$10,000) + .3(\$6,000) + .1(\$2,000) = \$8,000$$

Consider a random variable X . Its outcome is X_1 with a probability P_1 , X_2 with a probability P_2 , and so on. In general, the outcome is X_i with a probability P_i . Then the expected value of X is

$$E(X) = P_1X_1 + P_2X_2 + \dots + P_iX_i$$

Write this as

$$\text{Expected value of } X, \quad E(X) = \sum_{i=1}^n P_iX_i = \bar{X} \quad (1.7)$$

Next, we would like to know how much scatter, or dispersion, is present in this expected value of X . We may estimate this by the variance of X , or the standard deviation of X , defining them as follows.

$$\text{Variance of } X, \quad \text{var}(X) = \sum_{i=1}^n P_i(X_i - \bar{X})^2 \quad (1.8)$$

$$\text{Standard deviation of } X, \quad \sigma(X) = \sqrt{\text{var}(X)} \quad (1.9)$$

In the above example the standard deviation of the cash flow is

$$\sigma(X) = \sqrt{.6(10,000 - 8000)^2 + .3(6000 - 8000)^2 + .1(2000 - 8000)^2} = \$2683$$

This figure represents the uncertainty, or the margin of error, in the cash flow.

At times, it is necessary to find the mutual dependence of two different events. For example, we start two separate projects X and Y . The following table shows their expected cash flows. The first project X , is the same as the one discussed above on the previous page.

State of the Economy	Probability	Cash Flow X	Cash Flow Y
Good	60%	\$10,000	\$12,000
Fair	30%	\$6,000	\$8,000
Poor	10%	\$2,000	\$6,000

The two projects seem to be in step, both making more money in good economy and less in poor economy. They seem to be closely related. Is there a way to measure it quantitatively? The answer is yes, by using a measure called the correlation coefficient. First we define the covariance between two random variables X and Y as the

$$\text{Covariance between } X \text{ and } Y, \quad \text{cov}(X,Y) = \sum_{i=1}^n P_i(X_i - \bar{X})(Y_i - \bar{Y}) \quad (1.10)$$

To find the covariance between the two projects, we must first find the expected value of Y . Do it as

$$\bar{Y} = .6(\$12,000) + .3(\$8,000) + .1(\$6,000) = \$10,200$$

Next, we find

$$\begin{aligned} \text{cov}(X,Y) &= .6(10,000 - 8000)(12,000 - 10,200) \\ &\quad + .3(6,000 - 8,000)(8,000 - 10,200) + .1(2,000 - 8,000)(6,000 - 10,200) \\ &= 6,000,000 \end{aligned}$$

The six-million figure found above is not particularly meaningful. We next introduce a more practical measure of interdependence of two projects, the *correlation coefficient*, defined as

$$r(X,Y) = \frac{\text{cov}(X,Y)}{\sigma(X)\sigma(Y)} \quad (1.11)$$

Write the above equation as

$$\text{cov}(X,Y) = r(X,Y)\sigma(X)\sigma(Y) \quad (1.12)$$

We already know $\sigma(X)$ to be \$2683. We also evaluate $\sigma(Y)$ to be

$$\sigma(Y) = \sqrt{.6(12,000 - 10,200)^2 + .3(8000 - 10,200)^2 + .1(6000 - 10,200)^2} = \$2272$$

The smaller value of $\sigma(Y)$ indicates that the cash flows are more tightly bunched. Finally, we find the correlation coefficient as

$$r(X,Y) = \frac{\text{cov}(X,Y)}{\sigma(X)\sigma(Y)} = \frac{6,000,000}{2683*2272} = .9843 \heartsuit$$

Note that $r(X,Y)$ is a pure number and its value always lies between +1 and -1. That is

$$-1 < r(X,Y) < 1 \quad (1.13)$$

If the two projects are completely (meaning 100%), positively (meaning, moving in the same direction) correlated, the correlation coefficient between them is +1. This will be the case if one project is a carbon copy of the other one. If they are totally unrelated, the coefficient should be 0. This will be the case if one project is completely independent of the other one. If the two projects are such that whatever happens in one, the exact opposite happens with the other, then their correlation coefficient is -1.

The high value of $r(X,Y)$, .9843, in the above example is not particularly surprising because the two projects go hand in hand, performing well in good times and poorly in bad times. Some of these ideas are particularly helpful in understanding the risk and return of different portfolios.

1.5 Excel

It is important that the students are able to set up finance problems using Excel, which is now a standard of business and industry. A good working knowledge of this software should be an integral part of every business student's education. Almost all business programs offer courses in the use of this software. If you want to brush up your skill in the use of Excel, you may go the following Microsoft website for a variety of tutorials.

<http://office.microsoft.com/en-us/training/CR100479681033.aspx>

To get started on Excel, consider one of the previous problems that we solved by using the logarithm function.

1.2. Solve for x : $1.113^x = 2.678$

Set up the table shown below. Adjust the number in the **green** cell B2 until the numbers in cells B3 and B4 come very close together. B2 gives the answer.

	A	B
1	Base =	1.113
2	Unknown power =	9.201184226
3	Result (given) =	2.678
4	Result(calculated) =	=B1^B2

It is possible to embed an Excel table within a Word document. To do that, go the **Insert** tab in a Word document. When it opens, click on **Table**. In the **Table** menu, click on **Excel Spreadsheet** near the bottom. An Excel sheet opens up, where you can do your work. When you finish your Excel work, click anywhere on the Word document, and you can leave Excel. To go back into the Excel spreadsheet, double-click on the table, which will reveal all the calculations and formulas.

Next, consider example 1.8 on page 8 again. Set it up on Excel as follows.

	A	B	C	D
1	State of the Economy	Probability	Cash Flow X	Cash Flow Y
2	Good	60%	10000	12000
3	Fair	30%	6000	8000
4	Poor	10%	2000	6000
5	E(X)	=B2*C2+B3*C3+B4*C4	8000	
6	E(Y)	=B2*D2+B3*D3+B4*D4	10200	
7	Cov(X,Y)	=B2*(C2-B5)*(D2-B6)+B3*(C3-B5)*(D3-B6)+B4*(C4-B5)*(D4-B6)	6000000	
8	sigma(X)	=SQRT(B2*(C2-B5)^2+B3*(C3-B5)^2+B4*(C4-B5)^2)	2683.28157	
9	sigma(Y)	=SQRT(B2*(D2-B6)^2+B3*(D3-B6)^2+B4*(D4-B6)^2)	2271.56334	
10	r(X,Y)	=B7/B8/B9	0.98437404	

The numerical results of the formulas in cells B5:B10 are given in **green** cells C5:C10. The principal advantage of Excel is that it can handle large tables of numbers.

1.6 [Video 01C](#) Maple

[Maple](#) is an extremely powerful analytical software. Working with Maple is quite easy. The help facility in Maple is very valuable and it can guide the user through various steps, using plenty of examples. Maple has extensive application in science, mathematics, engineering, and finance. Time spent in learning this program can pay rich dividends in terms of greater accuracy and higher productivity. The following instructions will get you started with Maple.

Since Maple interprets capital and lower case letters distinctly, we should use the symbols carefully. Maple has many built in mathematical functions and constants, such as

ln, exp, Pi, sin, sqrt

Maple can do exact arithmetic calculations and displays the answer in its totality. For example, we need the exact value of 2^{64} , or the factorial of 50, or the value of π to 50 significant figures. We do this as follows: enter the commands at the > prompt, end each line with a semicolon, and strike the return key.

2^64;

18446744073709551616

50!;

30414093201713378043612608166064768844377641568960512000000000000

evalf(Pi,50);

3.1415926535897932384626433832795028841971693993751

Here

evalf

calculates the result in floating point with 50 significant figures. Maple can also do algebraic calculations. For instance, to solve the equations

$$\begin{aligned}5x + 6y &= 7 \\6x + 7y &= 8\end{aligned}$$

for x and y , enter the instructions as follows:

```
eq1 := 5*x+6*y=7;
eq2 := 6*x+7*y=8;
solve({eq1, eq2}, {x, y});
```

$$\begin{aligned}eq1 &:= 5x + 6y = 7 \\eq2 &:= 6x + 7y = 8 \\&\{y = 2, x = -1\}\end{aligned}$$

The symbol `:=` is used specifically to *define* objects in Maple. In other words, if we type

```
eq1;
```

then the computer will recall the equation defined as **eq1** and display it as

$$5x + 6y = 7$$

Maple can also do differentiation and integration. Consider the function

$$x^3 + \frac{\ln x}{x}$$

To differentiate this function with respect to x , we type in

```
diff(x^3+ln(x)/x, x);
```

$$3x^2 + \frac{1}{x^2} - \frac{\ln(x)}{x^2}$$

To integrate the result with respect to x , recreating the original function, we enter

```
int(%, x);
```

$$x^3 + \frac{\ln x}{x}$$

Here we use `%` as a symbol to designate the previous expression.

We can also use Maple to plot functions. For instance, if we want to see the visual representation of the well-known sine wave, as in Figure 1.1, we write.

```
plot(sin(x), x=0..2*Pi);
```

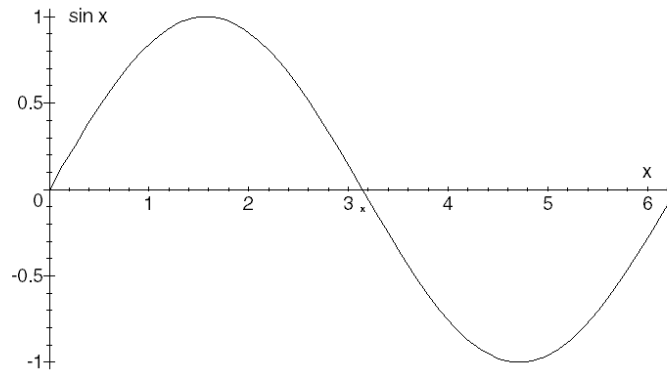


Fig. 1.1: Plot of $\sin x$ for $0 < x < 2\pi$

It is possible to add text in the plots, draw three-dimensional or animated plots, and draw plots in color. All plots in this book are drawn with the help of Maple.

Problems

Solve the following equations:

1.9. $16x - 54 = 15x - 32$ $x = 22$ ♥

1.10. $(x+1)(x-2) = (x-1)(x+2)$ $x = 0$ ♥

1.11. $(10x+3)(3x+4) = (5x+6)(6x+7)$ $x = -15/11$ ♥

1.12. $\frac{x-2}{x-3} = \frac{x-7}{x-9}$ $x = -3$ ♥

1.13. $\frac{x+4}{x+5} = \frac{x+6}{x+8}$ $x = -2$ ♥

Solve the following equations for x and y :

1.14.
$$\begin{aligned} 2x + 6y &= 32 \\ 5x + 8y &= 45 \end{aligned}$$
 $x = 1, y = 5$ ♥

1.15.
$$\begin{aligned} 3x + 4y &= 15 \\ 5x + 8y &= 45 \end{aligned}$$
 $x = -15, y = 15$ ♥

1.16. At Wal-Mart, in the hardware department, a customer buys five gallons of paint and six brushes and pays \$97.52 for them, including 6% sales tax. Another person buys eight gallons of paint and five brushes and pays \$146.28, including the sales tax. Find the price of a gallon of paint and that of a brush. Paint, \$16 per gallon; brushes, \$2 each ♥

Solve for x ,

1.17. $(1 + x)^{3.2} = 8.4$ $x = 0.9446$ ♥

1.18. $1.767^x = 3.876$ $x = 2.38$ ♥

1.19. $3.909^x = 15.99$ $x = 2.033$ ♥

Find the roots of

1.20. $2x^2 + 7x - 9 = 0$ $x = 1, -9/2$ ♥

1.21. $3x^2 + 4x - 7 = 0$ $x = 1, -7/3$ ♥

Find the sum of the following series:

1.22. $2.5 + (2.5)(.3) + (2.5)(.3)(.3) \dots$, infinite terms 3.571 ♥

1.23. $\frac{1}{1.1} + \frac{1}{1.1^2} + \frac{1}{1.1^3} + \dots$ 9 terms 5.759 ♥

1.24. $\frac{30}{1.12} + \frac{30(1.05)}{1.12^2} + \frac{30(1.05)^2}{1.12^3} + \dots$ 36 terms 386.60 ♥

1.25. $\sum_{i=1}^{10} \frac{500}{1.12^i}$ 2825.11 ♥

1.26. $\sum_{i=1}^{100} \frac{25}{1.12^i}$ 208.33 ♥

1.27. Write [WolframAlpha](#) instruction to find the sum, $\sum_{i=1}^{24} \frac{300}{1.01^i}$
`sum(300/1.01^i, i=1..24)`, 6373.02 ♥

1.28. The cash flows from two projects under different states of the economy are as follows:

State of the economy	Probability	Project A	Project B
Poor	20%	\$3000	\$5000
Average	30%	\$4000	\$7000
Good	50%	\$6000	\$15,000

Find the coefficient of correlation between the two projects. $.9922$ ♥

1.29. The expected return from two stocks, Microsoft and Boeing, under different states of the economy are as follows:

State of the economy	Probability	Microsoft	Boeing
Poor	10%	-5%	-40%
Average	40%	10%	-10%
Good	50%	20%	50%

- (A) Find the expected return of Microsoft and of Boeing. 13.5%, 17% ♥
- (B) Find the σ of Microsoft and of Boeing. 7.762%, 34.07% ♥
- (C) Find the coefficient of correlation between the two stocks. .9471 ♥

Key Terms

Accounts payable, 1	geometric series, 1, 6, 7	perpetuity, 7
Accounts receivable, 1	Inventories, 1	Preferred stock, 1
Accruals, 1	linear equation, 1	probability distribution, 9
annuity, 7	linear equations, 3, 5	quadratic equation, 1, 5
Cash, 1	Long-term bonds, 1	Retained earnings, 1
Common stock, 1	Maple, 1, 3, 12, 13, 14	standard deviation, 10
correlation coefficient, 10, 11	Marketable securities, 1	statistics, 1, 3, 8
covariance, 10	normal probability	
Excel, 1, 3, 11, 12	distribution, 9	
expected value, 9, 10	Notes payable, 1	