

3. OPTION VALUATION

Objective: After reading this chapter, you will understand the valuation of options.

3.1 Option Valuation

In this section, we will examine some of the basic concepts of option valuation. Later, we will use more precise valuation methods such as the [Black-Scholes formula](#) or the [binomial option-pricing model](#).

There are two types of options: the [European options](#), which can be exercised only at expiration, and the [American options](#), which may be exercised any time prior to expiration. The American option offers greater flexibility and hence its value, in general, is greater than the European one. We shall see the difference in their valuation a little later.

At this point, we are examining options on stocks that are not paying any dividends. When a stock pays a dividend then the value of the stock drops on the *ex dividend* date. This predictable drop in the price of a stock will have an effect on the price of the options on that stock. We shall look at the problem of dividend-paying stocks later.

Finally, we are not taking into consideration the transaction costs. Since the commission costs for trading options can be considerable, some of the results obtained in this section will appear to be violated in real life.

3.2 Properties of Option Values

1. The minimum value of an option is zero.

This is because an option is only a choice, not an obligation. The value of an option cannot be negative, because you do not have to do anything to get rid of it. The option will always have a zero, or a positive value.

2. The maximum value of a call option is equal to the value of the underlying asset.

This makes a lot of economic sense. An option allows you to buy a given asset at a certain exercise price. The most valuable option will be the one that allows you to acquire the asset at no cost, and the value of this option will be equal to the value of the underlying asset.

3. The total value of an option is the sum of its intrinsic value and its time premium.

$$\boxed{\text{Total value of an option}} = \boxed{\text{Intrinsic value of the option}} + \boxed{\text{Time premium of the option}}$$

The intrinsic value of an option is the value, or benefit, obtained by the holder by exercising the option immediately. The time premium of the option is its value, or benefit, of being able to wait and see. At expiration, the ability to wait is not there and so the time value of the option becomes zero.

For example, when a stock is selling for \$60 a share, its call option with exercise price \$55 is selling for \$8. Then the intrinsic value of the call is \$5 and the time value \$3. For another option priced at \$3 with stock price \$79 and exercise price \$80, the intrinsic value is zero, and hence the time premium is \$3.

The intrinsic value of a call option equals the difference between the stock price and the exercise price, if the stock price is higher; or the intrinsic value is zero if the stock price is less than the exercise price. We may express it as

$$\begin{aligned} \text{Intrinsic value of a call option} &= S - X, \text{ if } S > X \\ &= 0, \quad \text{if } S \leq X \end{aligned} \quad (3.1)$$

These two equations as a single equation,

$$\text{Intrinsic value of a call} = \max [S - X, 0] \quad (3.2)$$

The value of a put option increases as the stock price drops. This enables us to write

$$\text{Intrinsic value of a put} = \max [X - S, 0] \quad (3.3)$$

An option has time value only before its expiration. You lose the time value of an option when you exercise it before its expiration. Therefore, generally, it is not desirable to exercise an option before maturity. There is an important exception to this rule when we are evaluating the call options on dividend-paying stocks. When a stock pays cash dividends its price drops on the ex-dividend date, and the value of the call drops proportionately. If the time value of the option is less than the present value of the dividends paid during the life of the option, then it is better to forgo the time value of the option and capture the present value of the dividends.

4. At expiration, the value of a call option is

$$\begin{aligned} C_T &= 0, \quad \text{if } S_T \leq X \\ &= S_T - X, \text{ if } S_T > X \end{aligned} \quad (3.4)$$

where S_T is the stock price at time T when the option matures, and X is the exercise price. Another way of expressing this is

$$C_T = \max [0, S_T - X] \quad (3.5)$$

The value of an option is zero if one cannot derive any economic benefit out of it. This is the case when the final stock price is less than the exercise price. One could buy the stock

directly at a cheaper price and not use the call option. If we are not going to use an option, its value is zero. On the other hand, if the stock price is greater than the exercise price, then it is desirable to use the option and get the stock at a lower price, equal to the exercise price. The net advantage of using the option is just the difference between the final stock price and the exercise price.

For a put option, (3.5) becomes

$$P_T = \max [0, X - S_T] \quad (3.6)$$

5. In general, the value of an American option is higher than that of a European option.

One may exercise an American option at any time before expiration, but one can exercise a European option at its expiration only. This means that an American option offers greater flexibility to the option holder. A European is more restrictive, because you can utilize it only at its expiration. Thus, an American option is more desirable.

6. The maximum value of an American put is equal to its exercise price X .

We may exercise an American put at any time before expiration, allowing us to sell the stock at the exercise price. Therefore, the maximum amount that a put can realize is the exercise price.

7. The maximum value of a European put equals Xe^{-rT} .

An investor can exercise a European option only at expiration. Then it can fetch at the most X . The present value of that X is $X(1 + r)^{-T}$, where r is the interest rate and T is the time to expiration. If we think of time to be a continuous variable, and r to be the continuously compounded rate of interest, then we may write the above expression as

$$\lim_{n \rightarrow \infty} X(1 + r/n)^{-nT} = Xe^{-rT} \quad (3.7)$$

If the time to maturity of a European put is infinity, then the present value of X is zero. Therefore, the value of a European put with a very long time to maturity is close to zero.

8. A call with a higher exercise price has lower value compared to a similar call with lower exercise price.

$$C(S, X_2, T) \leq C(S, X_1, T), \text{ for } X_2 > X_1 \quad (3.8)$$

At expiration, the calls are in the money, if the stock price is higher than the exercise price. They are out of the money when the stock price ends up being less than the exercise price. For the call option to be of any value, the stock price must cross over the exercise price hurdle. Before maturity, the higher the exercise price, the lesser is the probability of jumping over it. Thus, the higher exercise price option is less valuable.

9. For European calls C_E , with different exercise prices, X_2 and X_1 , the relationship between call prices is

$$(X_2 - X_1) e^{-rT} \geq C_E(S, X_1, T) - C_E(S, X_2, T) \quad (3.9)$$

At expiration, the difference in the value of the options can be zero if they are both out of money and at most $(X_2 - X_1)$ if they are both in the money. Thus at expiration, $(X_2 - X_1)$ is always greater than or equal to the difference in the values of the options. The present value of this quantity, $(X_2 - X_1) e^{-rT}$ is always greater than the difference between the present values of the two options. This proves the result.

For American options C_A , the following relationship holds

$$(X_2 - X_1) \geq C_A(S, X_1, T) - C_A(S, X_2, T) \quad (3.10)$$

This is because we may exercise the American options at any time before maturity and realize their present value immediately.

Examples

3.1. Show that before maturity, the value of a call option is

$$C \geq \max [0, S - X e^{-rT}] \quad (3.11)$$

To prove this proposition, let us make a portfolio by buying a call at C , selling a share of stock at S , and also buying a bond whose face amount is X . The bond will mature after time T , at the same time when the option matures. We buy the bond at a discount, and so its present value is $X(1 + r)^{-T}$. For continuous time, the present value is $X e^{-rT}$. The initial investment in the portfolio is

$$V_0 = C - S + X e^{-rT}$$

At expiration, the value of the bond is its face amount, X . In addition, at expiration, if the option is in money, then the final value of the portfolio is

$$V_T = S_T - X - S_T + X = 0$$

If the final stock price is less than or equal to the exercise price, the option is out of money, and its value is zero. The total value of the portfolio is then

$$V_T = 0 - S_T + X \geq 0$$

Since the final value of the portfolio is greater than or equal to zero, its initial value is also greater than or equal to zero. This means

$$C - S + X e^{-rT} \geq 0$$

Or,
$$C \geq S - X e^{-rT}$$

In order to preclude negative values of C , we write the above result as

$$C \geq \max [0, S - X e^{-rT}]$$

3.2. Show that an American call with a longer time to maturity is more valuable than an otherwise similar call with shorter maturity. That is,

$$C_2(T_2) \geq C_1(T_1), \text{ where } T_2 \geq T_1 \quad (3.12)$$

To prove this proposition, let us consider its converse. Suppose the longer-term call is cheaper than the shorter-term call. Then we make a portfolio by buying the cheaper call (with longer time) and selling the more expensive one. We may write the initial value of the portfolio as

$$V_0 = C_2(T_2) - C_1(T_1) \leq 0$$

Since the initial value is zero or less, setting up this portfolio will actually make money for us. We liquidate the portfolio when the first option matures after time T_1 . If the shorter-term option is out of the money, we simply sell the longer-term option and make some more money. The net value of the portfolio at expiration is positive. If the first option is in the money, and the buyer exercises it, then we deliver the stock by exercising the other option. This will not cost us anything, and the final value of the portfolio is zero. Now, it is impossible for a portfolio with negative initial value to have zero or positive final value. The converse being impossible proves the proposition.

3.3. Consider the following options on Google, trading on May 23, 2008. The options were to expire on June 21, 2008. The time to maturity is thus 29 days. The closing stock price for the day was 544.62, with a change -4.84 . The risk-free rate was 1.85%.

Exercise price	Calls	Intrinsic value	Time value	Puts	Intrinsic value	Time value
510.0	41.30	34.62	6.68	5.27	0	5.27
520.0	34.20	24.62	9.58	8.30	0	8.30
530.0	27.10	14.62	12.48	11.50	0	11.50
540.0	21.20	4.62	16.58	15.50	0	15.50
550.0	16.30	0	16.30	20.60	5.38	15.22
560.0	12.50	0	12.50	26.80	15.38	11.42
570.0	9.00	0	9.00	33.30	25.38	7.92

The intrinsic and time value of the options have been calculated and added to the table. Now we make the following observations

1. The intrinsic value of an option can be positive, or zero.

2. The time value of an option is greatest when the exercise price is close to the stock price. This is a direct consequence of Black-Scholes model, which we study in the next chapter.

3. The time value of a call is more than the time value of a corresponding put. This comes out of Black-Scholes model as well. A common sense approach is that investors expect the stock price to rise. This is also true according to the capital asset pricing model. Thus they are willing to pay more for a call.

Exercise

3.4. Consider the following table of IBM option values on May 23, 2008. The closing stock price was 124.20. The options were to expire in June and October.

Expiration Date: June 21, 2008. Days to maturity: 29

Exercise price	Calls	Intrinsic value	Time value	Puts	Intrinsic value	Time value
110.0	14.90			0.20		
115.0	10.00			0.45		
120.0	5.80			1.23		
125.0	2.53			3.10		
130.0	0.76			6.30		
135.0	0.20			10.70		

Expiration Date: October 18, 2008. Days to maturity: 148

Exercise price	Calls	Intrinsic value	Time value	Puts	Intrinsic value	Time value
110.0	18.17			2.80		
115.0	14.07			4.20		
120.0	10.80			5.90		
125.0	7.90			8.10		
130.0	5.60			10.90		
135.0	3.60			13.70		

Fill in the intrinsic and time values. Use a calendar to find the number of days to maturity for these options.

- What relationship can you infer between the time value of a call option (or a put option) and the time to maturity?
- What is the relationship between the intrinsic value and the exercise price?
- Do you see any arbitrage opportunities in these options?