2. TIME VALUE OF MONEY

Objectives: After reading this chapter, you should be able to
1. Understand the concepts of time value of money, compounding, and discounting.
2. Calculate the present value and future value of various cash flows using proper mathematical formulas.

2.1 Single-Payment Problems

If we have the option of receiving $100 today, or $100 a year from now, we will choose to get the money now. There are several reasons for our choice to get the money immediately.

First, we can use the money and spend it on basic human needs such as food and shelter. If we already have enough money to survive, then we can use the $100 to buy clothes, books, or transportation.

Second, we can invest the money that we receive today, and make it grow. The returns from investing in the stock market have been remarkable for the past several years. If we do not want to risk the money in stocks, we may buy riskless Treasury securities.

Third, there is a threat of inflation. For the last several years, the rate of inflation has averaged around 3% per year. Although the rate of inflation has been quite low, there is a good possibility that a car selling for $15,000 today may cost $16,000 next year. Thus, the $100 we receive a year from now may not buy the same amount of goods and services that $100 can buy today. We can avoid this erosion of the purchasing power of the dollar due to inflation if we can receive the money today and spend it.

Fourth, human beings prefer to get pleasurable things as early as possible, and postpone unpleasant things as much as possible. We can use the $100 that we receive today buy new clothes, or to go out for dinner. If you are going to get the money a year from now, you may also have to postpone all these nice things.

Then there is the uncertainty of not receiving the money at all after waiting for a year. People are risk-averse, meaning, they do not like to take unnecessary risk. To avoid the uncertainty, or the risk of non-payment, we would like to get the money as soon as possible.

Banks and thrift institutions know that to attract deposits from investors, they must offer some kind of incentive. This incentive, the interest, compensates the depositors for their inability to spend their money immediately. For instance, if the bank offers a 5% rate of interest to the depositors, the $100 today will become $105 after a year.

Let us look at the problem analytically. If we deposit a sum of money with the present value *PV* in a bank that pays interest at the rate *r*, then after one year it will become *PV*(1 + *r*). Let us call this amount its future value *FV*. We may write it as

$$FV = PV\,(1 + r)$$

We may also think of (1 + *r*) as a growth factor. Continuing this process for another year, compounding the interest annually, the future value will become

$$FV = [PV\,(1 + r)](1 + r) = PV\,(1 + r)^2$$

This gives the future value after two years. If we can continue this compounding for *n* years, the future value then becomes

$$FV = PV\,(1 + r)^n \qquad (2.1)$$

The above expression is valid for *annual* compounding. If we do the compounding quarterly, the amount of interest credited will be only at the rate *r*/4, but there will also be 4*n* compounding periods in *n* years. Similarly, for monthly compounding, the interest rate is *r*/12 per month and the compounding occurs 12*n* times in *n* years. Thus, the above equation becomes

$$FV = PV\,(1 + r/12)^{12n}$$

At times, it is necessary to find the present value of a sum of money available in the future. To do that we write equation (2.1) as follows:

$$PV = \frac{FV}{(1 + r)^n} \qquad (2.2)$$

This gives the present value of a future payment. Discounting is the procedure to convert the future value of a sum of money to its present value. Discounting is a very important concept in finance because it allows us to compare the present value of different future payments.

Equations (2.1) and (2.2) relate the following four quantities:

FV = the future value of a sum of money
PV = the present value of the same amount
r = the interest rate, or the growth rate per period
n = number of periods of growth

If we know any three of the quantities, we can always find the fourth one.

2.2 Multiple-Payment Problems

In many financial situations, we have to deal with a stream of payments, such as rent receipts, or monthly paychecks. An annuity represents such a series of cash payments, even for monthly or weekly payments. Another example of an annuity is that of a loan that you take out and then pay back in monthly installments. Many insurance companies give the proceeds of a life insurance policy either as a lump sum, or in the form of an annuity. A perpetuity is a stream of payments that continues forever. In this section, we will learn how to find the present value and the future value of an annuity.

If there is a cash flow C at the end of first, second, third... period, then the sum of discounted cash flows is given by

$$S = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots\ n \text{ terms} \tag{2.3}$$

Here S represents the present value of all future cash flows. We compare it to the standard form of geometric series

$$S = a + ax + ax^2 + ax^3 + \ldots + ax^{n-1} \tag{1.1}$$

We notice that the first term $a = \dfrac{C}{1+r}$, and the ratio between the terms $x = \dfrac{1}{1+r}$. We know its summation as

$$S_n = \frac{a\,(1 - x^n)}{1 - x} \tag{1.2}$$

This gives

$$S = \frac{\dfrac{C}{1+r}\left(1 - \dfrac{1}{(1+r)^n}\right)}{1 - \dfrac{1}{1+r}}$$

Multiplying the numerator and the denominator in the above expression by $(1 + r)$, we get, after some simplification,

$$S = \frac{C\,[1 - (1+r)^{-n}]}{r} \tag{2.4}$$

Using the sigma notation for summation, we may write (2.3) as

$$S = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots\ n \text{ terms} = \sum_{i=1}^{n} \frac{C}{(1+r)^i}$$

Thus, we get a very useful result, namely,

$$\sum_{i=1}^{n} \frac{C}{(1+r)^i} = \frac{C\,[1-(1+r)^{-n}]}{r} \tag{2.5}$$

WRA `Sum[C/(1+r)^i,{i,1,n}]`

For a perpetuity, $(1 + r)^{-\infty} = 0$, and from (2.5) we have

$$\sum_{i=1}^{\infty} \frac{C}{(1+r)^i} = \frac{C}{r} \tag{2.6}$$

WRA `Sum[C/(1+r)^i,{i,1,infinity}]`

Note that (1.2) is a completely general formula for the summation of geometric series. We can use it to find the future value of an annuity. Equations (2.5) and (2.6) are special cases of (1.2) and they are useful only for finding the present value of an annuity or a perpetuity.

To review, the problems in this section can have either a single payment or multiple payments. The problems can be either future value or present value problems. The following examples illustrate the use of the above equations.

Examples

2.1. *Single payment, future value?* You would like to buy a house that is currently on the market at $85,000, but you cannot afford it right now. However, you think that you would be able to buy it after 4 years. If the expected inflation rate as applied to the price of this house is 6% per year, what is its expected price after four years?

Here we know the present value of the house, $85,000. Its price is going to grow at the rate of 6% per year for four years. Using (2.1), we get

$$FV = PV\,(1 + r)^n = 85{,}000(1.06)^4 = \$107{,}311 \; ♥$$

2.2. *Single payment, future value?* Jack has deposited $6,000 in a money market account with a variable interest rate. The account compounds the interest monthly. Jack expects the interest rate to remain at 8% annually for the first 3 months, at 9% annually for the next 3 months, and then back to 8% annually for the next 3 months. Find the total amount in this account after 9 months.

The annual interest rates are 8% and 9%, or .08 and .09. They correspond to monthly rates at 0.08/12 and 0.09/12. We compound the growth for the nine months as

$$FV = 6{,}000(1 + 0.08/12)^3(1 + 0.09/12)^3(1 + 0.08/12)^3 = \$6{,}385.58 \; ♥$$

2.3. *Single payment, future value?* You decide to put \$12,000 in a money market fund that pays interest at the annual rate of 8.4%, compounding it monthly. You plan to take the money out after one year and pay the income tax on the interest earned. You are in the 15% tax bracket. Find the total amount available to you after taxes.
The monthly interest rate is .084/12 = .007. Using it as the growth rate, the future value of money after twelve months is

$$FV = 12000(1.007)^{12} = \$13{,}047.73$$

The interest earned = 13,047.73 – 12,000 = \$1047.73. You have to pay 15% tax on this amount. Thus after paying taxes, it becomes =1047.73(1 – .15) = \$890.57

Total amount available after 12 months = 12,000 + 890.57 = \$12,890.57 ♥

2.4. *Present value, interest rate?* You expect to receive \$10,000 as a bonus after 5 years on the job. You have calculated the present value of this bonus and the answer is \$8000. What discount rate did you use in your calculation?

To find the present value of a future sum of money, we use

$$PV = \frac{FV}{(1+r)^n} \qquad (2.2)$$

This gives

$$8000 = \frac{10000}{(1+r)^5}$$

Or,

$$(1+r)^5 = 10{,}000/8000 = 1.25$$

$$1 + r = (1.25)^{1/5} = 1.0456, \text{ and thus } r = 4.56\%$$ ♥

To solve the problem on an Excel sheet, enter the following instructions.

	A	B	C
1	Future value, \$	10000	
2	Available after	5	years
3	Its present value, \$	8000	
4	The *required* discount rate	=(B1/B3)^(1/B2)-1	

You may get the result by entering the following on WolframAlpha.

WRA **8000=10000/(1+r)^5**

2.5. *Single payment, interest rate?* You have borrowed \$850 from your sister and you have promised to pay her \$1000 after 3 years. With annual compounding, find the implied rate of interest for this loan.

The future value of the loaned money is FV = \$1000, while its present value is PV = \$850. The time for compounding is $n = 3$ years. The interest rate r is unknown.

Using

$$FV = PV(1+r)^n \tag{2.1}$$

We get

$$1000 = 850(1+r)^3$$

or,

$$(1000/850)^{1/3} = 1 + r$$

or,

$$1 + r = 1.0556672$$

which gives

$$r = 0.0557 = 5.57\% \ ♥$$

WRA `1000=850(1+r)^3`

To solve the problem with the help of Maple, write

```
fsolve(1000=850(1+r)^3)
```

with the result .05566719198, which is 5.57%, as before. Here we use the command **fsolve**, rather than **solve**, to get the answer in floating point.

2.6. *Single payment, interest rate?* You have borrowed \$10,000 from a bank with the understanding that you will pay it off with a lump sum of \$12,000 after 2 years. Find the annual rate of interest on this loan.

Here the future value is \$12,000, present value \$10,000, and $n = 2$. Use

$$FV = PV(1+r)^n \tag{2.1}$$

This gives

$$12{,}000 = 10{,}000\,(1+r)^2$$

Or,

$$r = \sqrt{\frac{12{,}000}{10{,}000}} - 1 = .09545 = 9.545\% \ ♥$$

2.7. *Single payment, interest rate?* Ampere Banking Corporation offers two types of certificates of deposit, each requiring a deposit of \$10,000. The first one pays \$11,271.60 after 24 months, and the second one pays \$12,139.47 after 36 months. Find their monthly-compounded rate of return.

Using

$$FV = PV(1+r)^n \tag{2.1}$$

We get for the first CD,

$$11{,}271.60 = 10{,}000(1 + R_1)^{24}$$

Solving for R_1, we get

$$R_1 = \left(\frac{11{,}271.60}{10{,}000}\right)^{1/24} - 1 = 0.005$$

Similarly working on the second CD, we get

$$R_2 = \left(\frac{12{,}139.47}{10{,}000}\right)^{1/36} - 1 = 0.0054$$

The first certificate gives a return of .5%, and the second one .54% per month. The second one is higher because the investor has to tie up the money for a longer period. ♥

2.8. *Single payment, time?* A bank account pays 5.5% annual interest, compounded monthly. How long will it take the money to double in this account?

If the present value is \$1, its future value is \$2. The bank is compounding monthly, thus the interest rate is 5.5/12 percent per month. Using (2.1),

$$FV = PV\,(1 + r)^n \tag{2.1}$$

we get $$2 = 1(1 + .055/12)^n$$

Taking logarithms of both sides, ln 2 = n ln(1.0045833),

or, $n = \dfrac{\ln(2)}{\ln(1.0045833)}$ = 151.58 months = approximately, 12 years and 8 months. ♥

One can do the above example by using Excel, as follows. Adjust the number in the blue cell, B3, until the quantity in cell B4 becomes very close to 2.

	A	B	C
1	Present value, \$	1	
2	Interest rate, r	.055	per year, compounded monthly
3	Time required	151.58	months
4	Future value, \$	B1*(1+B2/12)^B3	2

To do the problem with Maple, we enter

```
solve(2=(1+.055/12)^n)
```

The result is 151.5784326, or 152 months.

WRA `2=(1+.055/12)^n`

2.9. *Multiple payments, future value?* Suppose you deposit \$350 at the beginning of each month in an account that pays 6% annual interest, compounded monthly. Find the total amount in this account at the end of 25 months.

The monthly rate of interest is ½%, or 0.005. Consider the first deposit of \$350. Its future value after 25 months is $350(1.005)^{25}$. The second deposit is a month late; it has only 24 months to grow, and its final value is $350(1.005)^{24}$. In a similar way, we find that the last deposit has just one month to earn interest. Putting it all together, the following expression gives the total at the end of 25 months:

$$S = 350(1.005)^{25} + 350(1.005)^{24} + ... + 350(1.005)$$

This is a geometric series with $a = 350(1.005)^{25}$, and $n = 25$. The exponent of the factor (1.005) is decreasing. This implies that the multiplicative factor $x = 1/1.005$. Using (1.2),

$$S_n = \frac{a\,(1 - x^n)}{1 - x} \qquad (1.2)$$

we find

$$FV = \frac{350(1.005)^{25}(1 - 1/1.005^{25})}{1 - 1/1.005} = \$9{,}342.17$$ ♥

To find the answer on WolframAlpha, enter the following and click on approximate form.

WRA `Sum[350*1.005^i,{i,1,25}]`

2.10. *Future amount, installment payment?* In order to buy a house you want to accumulate a down payment of \$15,000 over the next four years. You can do that by putting a certain sum of money in a savings account on the first of every month for the next 48 months. The account credits interest every month at the annual rate of 6%. What is your required monthly deposit?

Suppose you put C dollars on the first of every month for the next forty-eight months. The annual interest rate is 6%; the monthly interest rate is thus ½%, or .005. After 48 months, the first deposit has grown to $C(1.005)^{48}$. The next deposit has only 47 months to grow, and its final value is $C(1.005)^{47}$. Continuing in this fashion, the final total value in the account is the sum of future values of all deposits. We may write this as

$$15{,}000 = C(1.005)^{48} + C(1.005)^{47} + ... + C(1.005)$$

This is again a geometric series with $a = C(1.005)^{48}$, $n = 48$, and $x = 1/1.005$. Using (1.2) again, we have

$$S_n = \frac{a\,(1 - x^n)}{1 - x} \qquad (1.2)$$

Or,

$$15{,}000 = \frac{C\,(1.005)^{48}(1 - 1/1.005^{48})}{1 - 1/1.005}$$

which gives $C = \$275.89$ ♥

WRA **15000=Sum[C*1.005^i,{i,1,48}]**

To do the problem on Excel, enter the following. Adjust the number in the blue cell, B3, until the number in cell B4 comes very close to $15,000.

	A	B
1	No. of months	48
2	*Annual* interest rate, *r*	.06
3	*Monthly* deposit needed, $	275.89
4	Future amount, $15,000	=B3*((1+B2/12)^B1-1)/(1-1/(1+B2/12))

2.11. *Future amount, time required?* You have just opened an IRA in which you plan to deposit $100 a month, at the beginning of every month. The IRA will pay 9% annually, with monthly compounding. Approximately, how long will it take you to accumulate $20,000 in this account?

This is a multiple-payment, future value problem. Here FV = $20,000, P = $100, $r = 0.0075$, and n is the unknown quantity. We may write the future value of this account as the sum of future value of each of the monthly deposits. The first deposit will accumulate interest for n months, the second deposit for $n - 1$ months, and so on. The last monthly deposit, made at the beginning of the month, will earn interest only for that month. This expressed as

$$20{,}000 = 100(1.0075)^n + 100(1.0075)^{n-1} + \ldots + 100(1.0075)$$

Using (1.2), and with $a = 100(1.0075)^n$, $n = n$, $x = 1/1.0075$, we can sum the above series as

$$20{,}000 = \frac{100(1.0075)^n(1 - 1/1.0075^n)}{1 - 1/1.0075}$$

Rearranging terms,

$$20{,}000(1 - 1/1.0075) = 100(1.0075)^n - 100$$

$$148.88337 = 100(1.0075)^n - 100$$

$$248.88337 = 100(1.0075)^n$$

Or,

$$1.0075^n = 2.4888337$$

Taking logarithms of both sides, we get

$$n \ln(1.0075) = \ln(2.4888337)$$

Or,

$$n = \ln(2.4888337)/\ln(1.0075) = 122.0305670 \text{ 122 months}$$ ♥

To solve the problem on WolframAlpha, enter the following

WRA **20000=Sum[100*1.0075^i,{i,1,n}]**

To do problem using Excel, follow these instructions. Adjust the number in the blue cell, B3, until the number in B4 becomes very close to $20,000.

	A	B	C
1	Mo. deposit, $	100	at the beginning
2	Interest rate, r	.09	comp. monthly
3	Time required	122.03	months
4	Final value, $20,000	=B1*((1+B2/12)^B3-1)/(1-1/(1+B2/12))	

To do the problem with Maple, key in

```
20000 = sum(100*1.0075^i,i=1..n);
solve(%);
```

It gives the answer 122.0305695, which is approximately 122 months.

2.12. *Loan amortization, payment?* Suppose you borrow $10,000 at the annual interest rate of 9%, and you are required to pay it back in 60 equal monthly installments, the first one is due at the end of the first month. How much is the monthly installment?

The basic financial principle in a loan amortization, or loan repayment, problem is:

The present value of a loan	=	The present value of all future payments

The present value of the loan is $10,000. Equation (2.5),

$$\sum_{i=1}^{n} \frac{C}{(1+r)^i} = \frac{C\,[1-(1+r)^{-n}]}{r} \qquad (2.5)$$

gives the present value of the installment payments. With C = monthly payment, n = 60, r = .09/12 = .0075, we get

$$10{,}000 = \frac{C\,(1-1.0075^{-60})}{0.0075}$$

or,

$$C = \frac{0.0075(10{,}000)}{1-1.0075^{-60}} = \$207.58 ♥$$

WRA `10000=Sum[C/(1+.09/12)^i,{i,1,60}]`

2.13. *Loan amortization, payment?* You plan to buy a Jaguar XJ for $28,000, but you have only $6,000 in cash. The bank will loan you the rest at the annual interest rate of 12%, with the payments spread over 60 months. Find your monthly payment.

Since you already have \$6,000, you need to borrow \$22,000. Equating the present value of the loan to the present value of the payments, *P*, we get

$$22{,}000 = \frac{P}{1.01} + \frac{P}{1.01^2} + \ldots + \frac{P}{1.01^{60}}$$

We can do the summation by using (2.5), with $r = .01$, and $n = 60$. This gives us

$$22{,}000 = \frac{P\,[1 - 1.01^{-60}]}{0.01}$$

which gives, $P = \$489.38$ per month ♥

WRA `22000=Sum[x/1.01^i,{i,1,60}]`

2.14. *Loan amortization, payment?* Suppose the price of a house that you are interested in buying is \$100,000 and you have your \$15,000 down payment handy. The bank will loan you the remaining \$85,000 at 8% annual interest for a 25-year term. Find your monthly payment.

Equate the present value of the loan to the present value of the payments. Using (2.5), write it as

$$L = \sum_{i=1}^{n} \frac{C}{(1+r)^i} = \frac{C\,[1 - (1+r)^{-n}]}{r} \qquad (2.5)$$

In this case, $L = 85{,}000$, $n = 300$, $r = .08/12$, and the monthly payment is C. Thus

$$85{,}000 = \frac{C\,[1 - (1 + 0.08/12)^{-300}]}{0.08/12}$$

Or, $$C = \frac{85{,}000(.08/12)}{1 - (1 + .08/12)^{-300}} = \$656.04$$ ♥

WRA `85000=Sum[C/(1+.08/12)^i,{i,1,300}]`

2.15. *Future payments, present value?* Ronald Wilson has won a million dollars in the state lottery that will pay him \$50,000 annually in 20 annual installments. He will get the first installment right now. Using a discount rate of 10% per year, find the present value of all these payments.

The present value of the immediate payment is \$50,000. Thus the present value of the all 20 payments is given as

$$PV = 50{,}000 + \sum_{i=1}^{19} \frac{50{,}000}{1.1^i}$$

Use (2.5),

$$PV = 50{,}000 + \frac{50{,}000\,[1 - 1.1^{-19}]}{0.1} = \$468{,}246 ♥$$

Thus, the million-dollar lottery is worth only $468,246 in current dollars.

WRA **x=50000+Sum[50000/1.1^i,{i,1,19}]**

2.16. *Loan amortization, interest rate?* You have a loan of $5000 that you have to pay in 7 annual installments of $1100 each, the first one at the end of the first year. What is the annual interest rate on the loan?

In this problem we have to equate the present value of the loan, $5,000, to the present value of 7 payments, each one being $1,100. Use (2.5)

$$L = \sum_{i=1}^{n} \frac{C}{(1+r)^i} = \frac{C\,[1 - (1+r)^{-n}]}{r} \tag{2.5}$$

and substitute L = 5000, C = 1100, and n = 7. This gives

$$5000 = \frac{1100[1 - (1+r)^{-7}]}{r}$$

We may solve the above equation by any one of the following methods:

1. Use Excel. Adjust the value of the quantity in the blue cell B4 until the quantity in cell B5 becomes equal to the amount of loan.

	A	B
1	Amount of loan, $	5000
2	Number of payments,	7
3	Each payment, $	1100
4	Required interest rate, *r*	0.121268
5	Loan paid off, $5000	=B3*(1-1/(1+B4)^B2)/B4

You can calculate the value of r if you copy and paste the following instruction in any blank Excel cell.

```
=RATE(7,-1100,5000,0)
```

2. Use Maple. To solve the above equation using Maple, we key in

```
fsolve(5000=1100*(1-1/(1+r)^7)/r)
```

This gives the result as .1212687404, that is, 12.13%.

3. WRA **5000=Sum[1100/(1+r)^i,{i,1,7}]**

which gives the answer as $r \approx .121269$

4. Use a financial calculator programmed to solve such problems.

5. Use PVIFA tables on page 150, Chapter 12, to solve the problem.

This is a somewhat archaic method to solve these problems, but you can still use it to get an approximate value of the implied interest rate. First, find the Present Value Interest Factor of an Annuity, or PVIFA, which is defined as

$$\text{PVIFA} = \frac{\text{Total value of the loan}}{\text{Amount of each installment}} = \frac{5000}{1100} = 4.5455$$

The number of installments, $n = 7$. If you move across the line for $n = 7$, searching for this number, you will find it between $r = 12\%$ and $r = 13\%$. Now, interpolate the value of r as follows:

n	*r*	PVIFA
7	12%	4.5638
	13%	4.4226
	Difference for 1%	0.1412

The difference between 4.5455 and 4.5638 is 4.5638 − 4.5455 = 0.0183. Thus a more precise value of r is 12% + (.0183/.1412) of 1%. This give r = 12.1296034 ≈ 12.13%

2.17. *Loan amortization, interest rate?* You are planning to buy a high definition TV set from your friend for $1200 cash. Alternatively, he would allow you to pay for it in six monthly installments of $210 each, the first one after one month. What is the implied monthly rate of interest in this transaction?

Equating the value of the loan to the present value of installment payments, we have

$$1200 = L = \sum_{i=1}^{n} \frac{C}{(1+r)^i} = \sum_{i=1}^{6} \frac{210}{(1+r)^i}$$

WRA `1200=Sum[210/(1+r)^i,{i,1,6}]`

and the result comes out as $r \approx 0.0141207$. This about 1.412% per month. ♥

2.18. *Loan amortization, interest rate?* You would like to buy an iPad from your friend who is asking $400 for it. However, you offer to pay for it in 3 monthly installments of $140 each, and you will pay the first $140 after one month. Find the implied *annual* interest rate in your offer.

Suppose the annual interest rate is r. The monthly rate is $r/12$. Equating the loan to the present value of installments, write

$$400 = \sum_{i=1}^{3} \frac{140}{(1 + r/12)^i}$$

WRA **400=Sum[140/(1+r/12)^i,{i,1,3}]**

This gives the answer $r \approx 0.297571$. The annual rate is 29.76% per year. ♥

2.19. *Loan amortization, time?* Suppose you have borrowed 72,000 as a mortgage loan on your house. The interest rate is 6%. The bank has calculated the monthly payment to be \$515.83. How long will it take you to pay the loan?

The monthly interest rate is ½%, or .005. In this problem, the number of payments, n, is unknown. Since the amount of loan is equal to the present value of the payments, we can write

$$72{,}000 = \sum_{i=1}^{n} \frac{515.83}{1.005^i}$$

Or,
$$72{,}000 = \frac{515.83(1 - 1.005^{-n})}{.005}$$

Or,
$$\frac{.005(72{,}000)}{515.83} = 1 - 1.005^{-n}$$

Or,
$$\frac{.005(72{,}000)}{515.83} - 1 = -\,1.005^{-n}$$

Or,
$$-.3020956517 = -\,1.005^{-n}$$

Canceling the negative signs,
$$.3020956517 = 1.005^{-n}$$

Taking logarithms,
$$\ln(.3020956517) = -\,n \ln(1.005)$$

Or,
$$-1.197011584 = -\,n(.004987541511)$$

Or,
$$n = \frac{1.197011584}{.004987541511} = 240 \text{ months} = 20 \text{ years.}$$ ♥

WRA **72000=Sum[515.83/1.005^i,{i,1,n}]**

2.20. *Comparing present values:* You want to buy a piece of land for \$12,000 cash. The owner would allow you to pay for it in six annual installments of \$2300 each, the first one right now. Which method is cheaper for you if the time value of money is 12%?

We must compare the present value of the two methods of payment. Choose the smaller one as the better one.

PV of installment payments = $2300 + \sum_{i=1}^{5} \frac{2300}{1.12^i} = 2300 + \frac{2300[1-1.12^{-5}]}{.12}$ = \$10,590.98

PV of cash payment = \$12,000, and thus it is cheaper to pay by installments. ♥

WRA `2300+sum[2300/1.12^i,{i,1,5}]`

Problems

2.21. Adams Company bought a piece of land in 1981 for \$200,000. By 2005, its value had increased to \$1 million. Find the annual rate of appreciation during this period.
6.936% ♥

2.22. Ahsan Co bought a piece of land in 1991 for \$160,000 which appreciated in value at the rate of 3% per year for the first three years and then at the rate of 4% for the next four years. Find its value after 7 years. \$204,534 ♥

2.23. Your employer has promised to give you a \$5,000 bonus after you have been working for him for 5 years. What is the present value of this bonus if the proper discount rate is 8%? \$3402.92 ♥

2.24. The U.S. government fixed the price of gold at \$35 an oz in 1934. In 2005, the price of the yellow metal was \$480 an oz. Calculate the price appreciation of gold as percent per year, compounded annually. 3.757% ♥

2.25. You expect to receive \$10,000 as a bonus after 6 years. You have calculated the present value of this bonus and the answer is \$7000. What interest rate did you use in your calculation? 6.125% ♥

2.26. A downtown bank is advertising that if you deposit \$1,000 with them, and leave it there for 65 months, you can get \$2,000 back at the end of this period. Assuming monthly compounding, what is the monthly rate of interest paid by the bank? 1.072% ♥

2.27. You decide to put \$10,000 in a money market fund that pays interest at the annual rate of 7.2%, compounding it monthly. You plan to take the money out after one year and pay the income tax on the interest earned. You are in the 25% tax bracket. Find the total amount available to you after taxes. \$10,558.18 ♥

2.28. Suppose you have decided to put \$200 at the beginning of every month in a savings account that credits interest at the annual rate of 6%, but compounds it monthly. Find the amount in this account after 30 years. \$201,907.52 ♥

2.29. Antioch Company is adding $25,000 per month to a pension fund. The fund will earn interest at the rate of 6% per year, compounded monthly. Find the amount available in this fund after 20 years. $11.609 million ♥

2.30. Cincinnati Company has decided to put $30,000 per quarter in a pension fund. The fund will earn interest at the rate of 6% per year, compounded quarterly. Find the amount available in this fund after 10 years. $1,652,457 ♥

2.31. Suppose you put $250 at the beginning of every month in a savings account that credits interest at the annual rate of 6%, but compounds it monthly. Find the amount in this account after 25 years. $174,114.73 ♥

2.32. Suppose you deposit $125 on the first of every month for 240 months, and the bank credits interest at the end of every month at the annual rate of 6%. How much money do you have in your account at the end of 20 years? $58,043.89 ♥

2.33. You have decided to put $130 in a savings account at the *end* of each month. The savings account credits interest monthly, at the annual rate of 6%. How much money is in your account after 6 years? $11,233.15 ♥

2.34. Fred Abbott has just opened an IRA in which he plans to deposit $150 at the *end* of every month. The account will compound interest monthly at the annual rate of 9%. How much money will Fred have after 25 years in this account? $168,168.29 ♥

2.35. You have started a job with an annual salary of $48,000. You will get the paycheck at the *end* of each month, and your deductions for taxes will be 34%. Using a discount rate of 0.8% per month, find the present value of the take home pay for the whole year.
$30,092.34 ♥

2.36. Suppose you want to accumulate $10,000 for a down payment for a house. You will deposit $400 at the beginning of every month in an account that credits interest monthly at the rate of 0.6% per month. How long will it take you to achieve your goal?
24 months. ♥

2.37. James Earl has decided to save a million dollars by depositing $50,000 at the beginning of each year in an account that pays interest at the rate of 10%, compounded annually. How long will it take him to reach his objective? 11 years ♥

2.38. Suppose you want to accumulate $25,000 as down payment on a house and the best you can do is to put aside $200 a month. If you deposit this amount at the beginning of each month in an account that credits 0.75% interest monthly, how long will it take you to attain your goal? 88 months ♥

2.39. Suppose you deposit $300 at the beginning of each month in a savings account that pays interest at the rate of 6% per year, with monthly compounding. How long will it take you to accumulate $25,000 in this account? 5 years, 10 months ♥

2.40. Emily Dickinson would like to accumulate $12,000 for a down payment on a house by depositing $400 on the first of every month in a savings account that pays 6% annual interest, compounded monthly. How long will it take her to reach her goal? 28 months ♥

2.41. Suppose you deposit $70.97 at the beginning of every month in an account that pays 9% interest per year, compounding it monthly. You would like to accumulate $10,000 in this account. How long do you have to wait before you reach your goal? 8 years ♥

2.42. Suppose you are a property owner and you are collecting rent for an apartment. The tenant has signed a one-year lease with $600 a month rent, payable in advance. Find the present value of the lease contract if the discount rate is 12% per year. $6820.58 ♥

2.43. Republic of Zimbabwe has borrowed $50 million from the World Bank at an interest rate of 3% per year. Zimbabwe will repay the loan over the next 30 years in equal annual payments. Find the annual installment. $2.551 million ♥

2.44. West Bank gives consumer loans at the annual interest rate of 8.25%. Suppose you take out a $5,200 loan for 36 months, what will your monthly payment be? $163.55 ♥

2.45. Easton Bank gives 6% annual interest, compounded monthly, on its savings deposits. Suppose you deposit $100 on the first of every month in the bank, how long will it take you to accumulate $10,000? 81 months ♥

2.46. You want to buy a $120,000 house, and you apply for a mortgage loan. The bank requires a 20% down payment. It will give you a 25-year loan at 8.75% annual interest rate, payable in monthly installments. How much is your monthly payment? $789.26 ♥

2.47. Adana Corporation is interested in buying a building for $500,000 in cash, or it may pay for it in 50 monthly installments of $12,000 each. If the proper discount rate for Adana is 9%, which method should it use? PV(installments) = $498,797.36, better ♥

2.48. Alhambra Corporation borrowed $1 million from Anaheim Bank with the understanding that Alhambra will pay the loan back in 6 monthly installments of $175,000 each. Find the annual rate of interest charged by the bank. 16.94% ♥

2.49. Akron Corporation has borrowed $1 million from Canton Bank with the understanding that Akron will pay the loan back in 12 monthly installments of $90,000 each. Find the annual rate of interest charged by the bank. 14.45% ♥

2.50. Armes Corporation has the opportunity to receive $20,000 right now, or, $3254.91 per year for the next ten years. The first payment will be available after one year. For what rate of interest would the two options be of equal value? 10% ♥

2.51. Auckland Corporation has borrowed $700,000 from a bank. Auckland will repay the loan in ten annual installments of $100,000 each. The first installment will be paid a year from now. Find the rate of interest charged by the bank. 7.07% ♥

2.52. Suppose you buy a machine and you have the option of paying the full price, $40,000, now; or $10,000 at the end of each of the next five years. What is the cost of capital, or the implied interest rate, for the two methods to be equivalent? 7.93% ♥

2.53. You have bought a car. The car dealer offers two payment plans: (A) Make 48 monthly payments of $130 each, or (B) Make 36 payments of $165 each. If the time value of money is 12% per year, which plan is cheaper for you? (A) by about $31 ♥

2.54. Bennington Company has borrowed a certain amount from the bank that it will repay in 24 monthly installments. The bank charges 6% interest annually on this loan and the monthly payment is $6000. Find the amount of loan. $135,377.20 ♥

2.55. You want to buy a piece of land and the owner would sell it to you for $20,000 cash. Alternatively, he would let you pay for it with five annual installments of $5,000 each, the first one due right now. What is the implied interest rate here? 12.59% ♥

2.56. Karl has borrowed $400 from his friend Bill and he will pay him back in four monthly installments of $105 each. Find the monthly rate of interest charged by Bill.
1.98% ♥

2.57. Dickens Corp wants to buy a 100-acre tract of land. The owner will sell it for a cash price of $175,000, but Dickens offered to pay for the land in five annual installments of $40,000 each, the first one is due at the end of one year. Find the cost of capital for Dickens the two prices to be equivalent. 4.62% ♥

2.58. Edinburgh Corporation has the choice of paying for a piece of land $5 million in cash now. Or, after making a down payment of $1 million, it may pay the balance may in 6 equal annual payments of $1 million. Find the implied rate of interest in the second option. 12.98% ♥

2.59. Suppose you have borrowed $12,000 from a bank with the interest rate of 11.5%. Your monthly installments are $313.07. How long will it take you to pay the loan?
48 months ♥

2.60. You have borrowed $10,000 from a bank at the interest rate of 1% per month. Your monthly payment is $554.15. Find the time required to repay the loan. 20 months ♥

2.61. Allegheny Company has borrowed $100,000 from a bank with the understanding that the company will pay $2,000 per month to repay the loan. The bank will charge .75% interest per month on the unpaid balance. How long will it take Allegheny to amortize the loan? 63 months ♥

2.62. Ames Corporation has borrowed $5 million from a bank with the understanding that it will pay the loan in monthly installments of $100,000 each. The bank charges interest at the rate of 0.8% per month. Find the time required to pay the loan.

65 months ♥

2.63. Louis Trichardt would like to save $15,000 to use as a down payment on a house. He will deposit $500 a month in a savings account that pays interest at the rate of 6% per year, compounded monthly. How long will it take him to accomplish his objective?

28 months ♥

2.64. Durban Corporation is interested in acquiring a machine that it can buy for $140,000 in cash. Alternatively, Durban can make five equal payments of $40,000 each, the first one due after one year, to purchase the same machine. Find the implied interest rate in the second option. 13.20% ♥

Multiple Choice Questions

1. The future value of $1100, compounding at the rate of 6% annually, after 10 years is

(a) $1600.00 (b) $1790.85
(c) $1819.40 (d) $1969.93

2. The present value of $5000 that you will get after 10 years, discounting at the rate of 5% per year, is

(a) $2508.91 (b) $2899.77
(c) $2965.34 (d) $3069.57

3. Suppose Republic of Scandia has a steady 30% inflation rate and a loaf of bread costs 100 liras today. Its price, in liras, last year was

(a) 66.67 (b) 70
(c) 76.92 (d) 130

4. The monthly interest rate on a savings account is 1%, compounded monthly. The effective annual rate is

(a) 11.25% (b) 12.00%
(c) 12.68% (d) 13.13%

5. If the discount rate is 7%, then the present value of $40,000 that you expect to get after 15 years is

(a) $14,497.84 (b) $15,037.48
(c) $106,400.80 (d) $110,361.26

6. The future value of $10,000 after 11 years, growing at the rate of 12% per year is

(a) $34,237.40 (b) $34,522.71
(c) $34,785.50 (d) $34,984.51

Key terms