

4. CAPITAL ASSET PRICING MODEL

Objectives: After reading this chapter, you should

1. Understand the concept of beta as a measure of systematic risk of a security.
2. Calculate the beta of a stock from its historical data.
3. Understand the Capital Asset Pricing Model.
4. Apply it to determine the risk, return, or the price of an investment opportunity.

4.1 Beta

In the section on capital budgeting, we saw the need for a risk-adjusted discount rate for risky projects. The risk of an investment or a project is difficult to measure or quantify. This difficulty arises from the fact that different persons have different perceptions of risk. What may be quite a risky project to one investor may appear to be fairly safe to another person. After all, how can you quantify courage, or patience, or risk, or beauty?

In the section on portfolio theory, we used σ as a measure of risk, which is really the standard deviation of returns. Another useful measure of risk is the β of an investment. Like σ , β is also a statistical measure of risk. We infer it from the observations of the past performance of a stock. For example, we may want to find the risk of buying and holding the stock of a particular corporation, such as IBM, and we are interested in finding the β of IBM. We can start by looking at the historical value of three variables:

1. *The returns of IBM stock, R_j .* We define the return on a stock by the relation

$$R_j = \frac{P_1 - P_0 + D_1}{P_0} \quad (4.1)$$

In the above equation, P_0 is the purchase price of the stock, P_1 its price at the end of the holding period, and D_1 is the dividend paid, if any, at the end. The quantity $P_1 - P_0$ is the price appreciation of the stock, and along with the dividend, is the total change in the value of the investment. The return is equal to be the change in the value of the investment divided by the original investment. For example to find the monthly rate of return on the IBM stock, we may want to know the price of the stock at the beginning of each month, the price at the end of the month, and the dividends paid during that particular month. We have to develop a series of numbers representing the return for each month for the last 24 months, say.

2. *The returns of the market, R_m .* A *market index* provides an overall measure of the performance of the market. The oldest and the most popular market index is the Dow Jones Industrial Average. The problem with this index is that it uses only 30 stocks in its valuation. For a broader market index, we may have to look at S&P100, or S&P500 index. There is even an index for over-the-counter stocks called the NASDAQ Composite Index. The value of these indexes is available daily.

Let us track the market for the last 24 months. If we know the value of the index at the start and finish of each month, we can find the return of the market for that month. The dividend yield for the market is around 1.71% annually at present. Therefore, we define the overall return on the market as

$$R_m = \frac{M_1 - M_0}{M_0} + d_1 \quad (4.2)$$

where M_0 is the beginning value and M_1 the ending value of the market index, and d_1 is the dividend yield as a percent for that period. With some effort, we may be able to develop a set of market returns for each of the last 24 months.

3. *The riskless rate of interest, r .* The securities issued by the Federal government, such as the Treasury bills, bonds, and notes, are, by definition, riskless. They are the safest investments available, backed by the full faith and taxing power of the government. Their rate of return depends on their time to maturity, and for longer maturity, the return is generally higher. The Treasury yield curve is available on the Internet.

After some research, we may also get a series of riskless rates for each of the past 24 months.

Then we define two variables x and y as:

$$\begin{aligned} y &= R_j - r \\ x &= R_m - r \end{aligned}$$

where R_j = return on the stock j each month for the last 24 months,
 R_m = corresponding monthly returns on the market for the same period,
 and r = riskless rate of interest per month, for the last 24 months.

By subtracting the riskless rate of interest, we are able to see the return due to the risk inherent in the given stock, and the return from the risk in the market. Thus, we are comparing the returns exclusively due to the risk in the investments.

A regression line drawn between the various observed values of x and y will show a certain linear relationship between x and y . The slope of the line will give the rate of change of y with respect to x . In other words, the slope will signify how much the return on the stock will change corresponding to a given change in the return on the market. In this diagram let us say that the slope of the line is β , and the y -intercept is α . The quantity α is practically zero, and it is statistically insignificant. The quantity β , on the other hand, represents an important concept.

This responsiveness of the stock return to the changing market conditions is called the "beta" of the stock. Stocks with low betas will show very little movement due to the fluctuations in the stock market. High beta stocks will tend to be jumpy showing a large variation in response to small changes in the market.

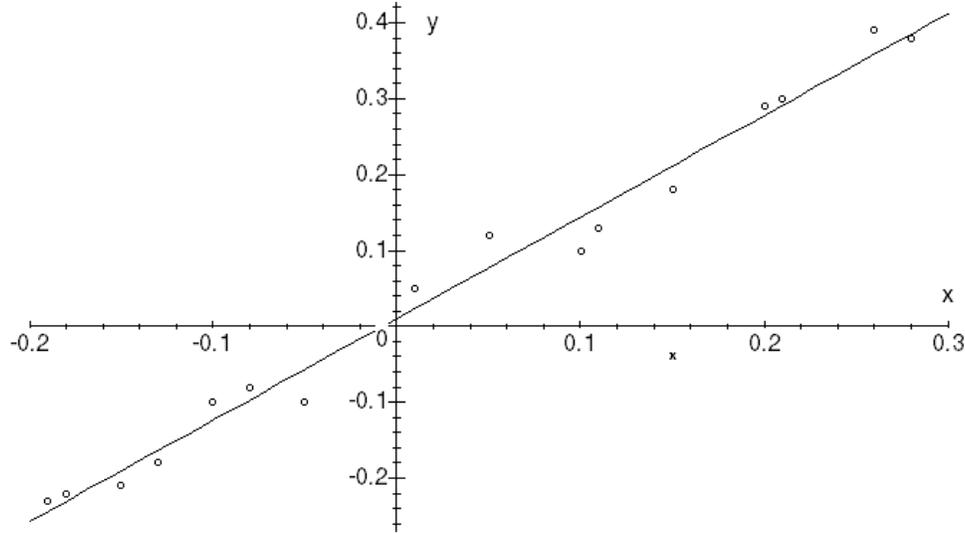


Fig. 4.1: A regression line between y and x .

High β stocks, due to their large volatility, will be more unpredictable, and therefore, more risky. Low beta stocks show relatively small volatility, and they are more predictable and safe.

Beta is a statistical quantity, and it is a measure of the systematic risk, or the market related risk of a stock. These results can also be expressed as a statistical formula,

$$\beta_j = \frac{\text{cov}(R_j, R_m)}{\text{var}(R_m)} = \frac{r_{jm}\sigma_m\sigma_j}{\sigma_m^2} = \frac{r_{jm}\sigma_j}{\sigma_m} \quad (4.3)$$

where $\text{cov}(R_j, R_m)$ is the covariance between the returns on the stock j and the market, and $\text{var}(R_m)$ is the variance of the returns on the market. If we have collected sufficient statistical data, we may find β by using

$$\beta = \frac{n\sum(xy) - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \quad (4.4)$$

$$\alpha = \frac{\sum y - \beta \sum x}{n} \quad (4.5)$$

where n is the number of x and y values.

One can apply the concept of beta to a portfolio. The beta of a portfolio is simply the weighted average of the betas of the securities in the portfolio,

$$\text{Beta of a portfolio, } \beta_p = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + \dots = \sum_{i=1}^n w_i\beta_i \quad (4.6)$$

The advantage of using β as a measure of risk is that it can combine linearly for different securities in a portfolio, but the disadvantage is that it can measure only the market related risk of a security. On the other hand, σ can measure the risk independent of the market conditions, but its disadvantage is that it is non-linear in character and difficult to apply in practice. Both β and σ are incomplete measures of risk; they change with time, and are difficult to measure accurately.

By definition, the beta of a riskless investment is zero. Further, the beta of the market is 1. This is seen by setting $j = m$ in (4.3) and noting that the covariance of a random variable with itself is just its variance.

A security that has a high beta should show a large rise in price when there is an upward movement in the market, and has a large drop in price in case of a downward movement. These large price fluctuations can cause a considerable amount of uncertainty about the return of this security, and greater risk associated with it. Therefore, a high beta security is also a high-risk security. Thus, beta is frequently used as a measure of the risk of a security. A low beta security is a defensive security and a high beta of a stock means a more aggressive management stance.

The numerical value of β for different stocks is available from sources on the Internet, such as www.etrade.com, and www.yahoo.com.

Examples

Video 04.01 4.1. Calculate the β of Hauck Corporation from the following data. The prices are at the beginning and end of each year:

Year	Price of Hauck	Dividend of Hauck	Market index	Market dividend	Riskless rate
2005	25–27	\$1.00	100–105	3.05%	6.00%
2006	27–29	\$1.00	105–110	3.00%	6.00%
2007	29–32	\$1.50	110–120	2.95%	5.95%
2008	32–33	\$1.50	120–125	2.80%	5.90%

The return from the security in 2005 is capital gains (\$2) plus dividends (\$1) divided by the initial price (\$25), that is, $3/25 = 0.12$. The riskless rate during 2005 was 0.06, thus the excess return was $0.12 - 0.06 = 0.06$. The return on the market for the same year was $5/100 + 0.0305 = 0.0805$. The excess return was $0.0805 - 0.06 = 0.0205$. Designating the excess return for security as y and that for the market as x , we can tabulate the calculations as:

Year	R_j	$-r$	$= y$	R_m	$-r$	$= x$
2005	3.00/25	-.06	= .06	5/100 + .0305	-.06	= 0.0205
2006	3.00/27	-.06	= .051111	5/105 + .03	-.06	= 0.017619
2007	4.50/29	-.0595	= .095672	10/110 + .0295	-.0595	= 0.060909
2008	2.50/32	-.059	= .019125	5/120 + .028	-.059	= 0.010667

Here $n = 4$, the number of periods, or x, y pairs

$$\begin{aligned}\Sigma xy &= (.0205)(.06) + (.017619)(.051111) + (.060909)(.095672) + (.010667)(.019125) \\ &= 0.0081618\end{aligned}$$

$$\Sigma x = 0.0205 + 0.017619 + 0.060909 + 0.010667 = 0.109695$$

$$\Sigma y = .06 + .051111 + .095672 + .019125 = 0.225908$$

$$\Sigma x^2 = (0.0205)^2 + (0.017619)^2 + (0.060909)^2 + (0.010667)^2 = 0.00455437$$

Using equation (4.4)

$$\beta = \frac{4(0.0081618) - (0.109695)(0.225908)}{4(0.00455437) - (0.109695)^2} = 1.271927967 \approx 1.27$$

One can do the above problem with the help of Maple as follows:

```
#n is the number of periods, or returns
#n+1 is the number of price data points
n:=4;
#Price is an array to store price of stock
Price:=array(1..n+1, [25,27,29,32,33]);
#Div is an array to store dividends
Div:=array(1..n, [1,1,1.5,1.5]);
#Market is the array to store market index data
Market:=array(1..n+1, [100,105,110,120,125]);
#Markdiv is the array to store market dividends as percent
Markdiv:=array(1..n, [.0305, .03, .0295, .028]);
#RF is the array to store riskfree rate
RF:=array(1..n, [.06, .06, .0595, .059]);
#x, y are the arrays to store x, y values
x:=array(1..n); y:=array(1..n);
for i to n do
x[i]:=(Market[i+1]-Market[i])/Market[i]+Markdiv[i]-RF[i];
y[i]:=(Price[i+1]-Price[i]+Div[i])/Price[i]-RF[i] od;
unassign('i');
n*sum(x[i]*y[i], i=1..n)-sum(x[i], i=1..n)*sum(y[i], i=1..n);
n*sum(x[i]^2, i=1..n)-sum(x[i], i=1..n)^2;
beta=%%/%;
```

4.2. Calculate the β of Maine Corporation from the following data. The prices are at the beginning and at the end of each year:

Year	Price of Maine	Dividend of Maine	S&P 500 index	S&P 500 dividend	Riskless rate
2000	25-27	\$2.00	100-105	3.05%	8.0%
2001	27-29	\$2.00	105-110	3.20%	8.5%
2002	29-32	\$2.50	110-120	3.50%	7.5%
2003	32-33	\$2.50	120-125	4.00%	7.0%

$$\beta = 0.89 \quad \heartsuit$$

4.2 Capital Asset Pricing Model

Beta is a measure of the market risk, or the systematic risk, of a security. A security with a large beta will have large swings in its price in relation to the changes in the market index. This will lead to a higher standard deviation in the returns of the security, which will indicate a greater uncertainty about the future performance of the security.

Draw a diagram with the β of various securities along the X-axis and their expected return along the Y-axis. We have already noticed that β is a linear measure of risk. If we *assume* that a linear relationship exists between the risk and return, then only two points are sufficient to draw a straight line in this diagram. The line, representing the relationship between risk and expected return, is called the *security market line*. Under equilibrium conditions, all other securities will also lie along this line. Higher β securities will have a correspondingly higher expected return. Figure 4.2 shows this graphically.

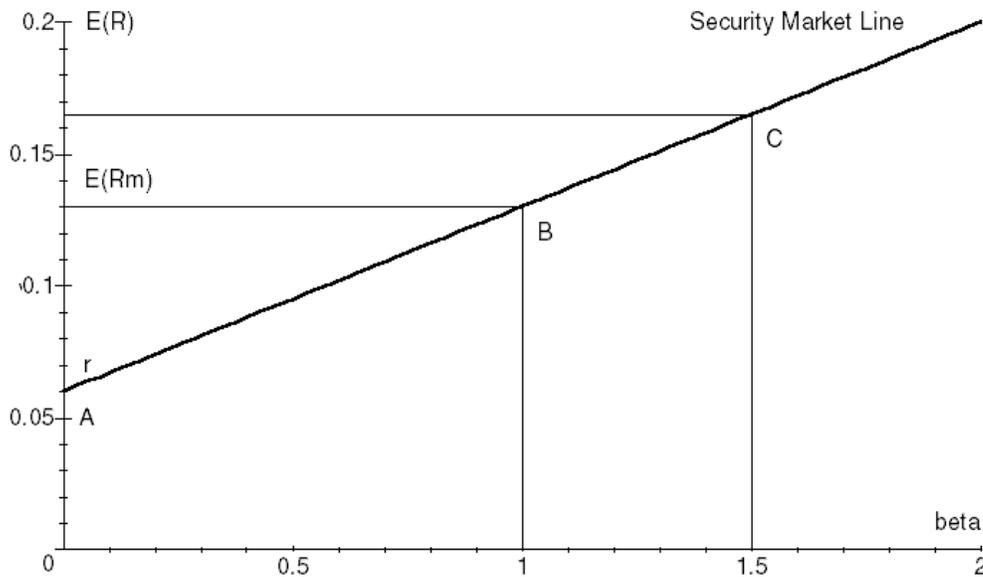


Fig. 4.2: Security market line.

By definition, beta of the market is equal to 1. The securities with more than average risk will have beta greater than 1, and less risky securities have beta less than 1. On this scale, the beta of a riskless security is zero. Such securities will provide riskless rate of return, r , to the investors. An example of such a security is the Treasury bill.

The security market line represents the risk-return characteristics of various securities, assuming that there is linear relationship between risk and return. Point A represents a riskless security with beta equal to zero and return r . Point B shows a market-indexed security which could be a very large mutual fund portfolio, which is invested in a large number of securities all weighted according to assets of the corporations whose securities make up the portfolio. Point C shows an individual security whose beta is β_i and whose expected return is $E(R_i)$. Since A, B, C all lie along the same straight line, then

Slope of segment AC = slope of segment AB

This gives,
$$\frac{E(R_i) - r}{\beta_i} = \frac{E(R_m) - r}{1}$$

Or,
$$E(R_i) = r + \beta_i [E(R_m) - r] \quad (4.7)$$



William Sharpe (1934-)

Equation (4.7) gives the expected return of a security i in terms of its risk, expected return on the market, and the riskless rate. It is a forward-looking model, and thus gives the expected values of the returns. This equation represents what is known as the "[Capital Asset Pricing Model](#)", CAPM for short, and was developed in the 1960s by [William Sharpe](#), [Jan Mossin](#), and [John Lintner](#). The use of this model is illustrated by the following examples.

Positive Alpha: Too Good to be True?

New research from Robert Jarrow suggests that positive alpha is improbable.

During the past 25 years, an entire segment of the investment industry was constructed on the belief that positive alphas exist and can be exploited by portfolio managers to yield greater profit at less risk. New research by the Johnson School's Robert Jarrow strongly suggests that positive alphas are rare to nonexistent.

"Every hedge fund in the world claims to have positive alpha, but I say it can't be," says Jarrow, Ronald P. and Susan E. Lynch Professor of Investment Management at the Johnson School. "The claims for positive alpha are too strong—professional investment managers are taking risks that are hidden."

Alpha, an estimate of an asset's future performance, after adjusting for risk, is a measure routinely calculated by portfolio managers. Positive alpha suggests that an investor can realize higher returns at lower risk than by holding an index. In other words, by investing in assets with positive alpha, one can "beat the market," without exposure to the risk otherwise associated with the promised rate of return.

Jarrow used mathematical modeling to prove that positive alphas are equivalent to arbitrage opportunities. And arbitrage opportunities—risk-free trading of an asset between two markets to take advantage of a price differential—are rare in financial markets. According to Jarrow's research, positive alpha can exist only in the presence of a true arbitrage opportunity. For this to occur, two stringent conditions must be met. First, there must exist a market imperfection that enables the arbitrage opportunity to persist, even as arbitrageurs capitalize upon it; second, there must be a source of financial wealth, on which the arbitrageurs draw, either knowingly or unknowingly.

"Academics have looked for arbitrage opportunities in financial markets, and haven't found many. So it seems implausible to have so many positive alphas out there." Jarrow says. "To have positive alpha for any length of time means that someone is consistently losing money to someone else, and that's hard to believe."

In his paper "Active Portfolio Management and Positive Alphas: Fact or Fantasy?" forthcoming in the Journal of Portfolio Management, Jarrow outlines his model and offers examples of both true and false positive alphas, drawn from the pivotal events of the credit market crisis. His conclusions include a word of caution to investors.

"The moral of this paper is simple," Jarrow writes. "Before one invests in an investment fund that claims to have positive alphas, one should first understand the market imperfection that is causing the arbitrage opportunity and the source of the lost wealth. If the investment fund cannot answer those two questions, then the positive alpha is probably fantasy and not fact."

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Examples

Video 04.03 4.3. Chicago Corp stock will pay a dividend of \$1.32 next year. Its current price is \$24.625 per share. The beta for the stock is 1.35 and the expected return on the market is 13.5%. If the riskless rate is 8.2%, what is the expected growth rate of Chicago?

Using the capital asset pricing model (CAPM),

$$E(R_i) = r + \beta_i [E(R_m) - r] \quad (4.7)$$

We first find the expected rate of return as

$$E(R_i) = 0.082 + 1.35 [0.135 - 0.082] = 0.15355 = R$$

The expected rate of return $E(R_i)$, for a security is also its required rate of return R by the investors. Using the growth model for a stock, equation (3.6),

$$P_0 = \frac{D_1}{R - g}$$

we get, $R - g = D_1/P_0$, or $g = R - D_1/P_0$,

which gives $g = 0.15355 - 1.32/24.625 = 0.1$. Thus the growth rate is 10%. ♥

Video 04.04 4.4. Peggotty Services common stock has a $\beta = 1.15$ and it expects to pay a dividend of \$1.00 after one year. Its expected dividend growth rate is 6%. The riskless rate is currently 12%, and the expected return on the market is 18%. What should be a fair price of this stock?

$$E(R_i) = r + \beta_i [E(R_m) - r] \quad (4.7)$$

we get $E(R_i) = 0.12 + 1.15 [0.18 - 0.12] = 0.189$

Thus, the expected return on the stock is 0.189, and the expected growth rate is 0.06. Using (3.1) once again,

$$P_0 = \frac{1}{0.189 - 0.06} = \$7.75 \heartsuit$$

Video 04.05 4.5. The beta of Vega Inc is 1.15, its rate of growth is 10%, it will give a dividend of \$3.00 next year, and its common stock sells for \$50 a share. The riskless rate is 8%. By careful planning and by selecting more secure projects, Vega has reduced its risk. Its new beta is estimated to be 1, while everything else (income, dividends, growth rate, capital structure, market return, etc.) is the same. What is its new share value?

The total return on a stock is the sum of its dividend return and the growth rate. If r is the required rate of return, $E(R_i)$ is the expected rate of return, g is the growth rate, D_1 is the dividend to be paid next year, and P_0 is its price now, then

$$R = \frac{D_1}{P_0} + g = \frac{3}{50} + 0.1 = 0.16 = E(R_i)$$

Use $E(R_i) = r + \beta_i [E(R_m) - r]$ (4.7)

Drop the subscript i , and solve for $E(R_m)$, to get

Or,
$$E(R_m) = r + \frac{E(R) - r}{\beta}$$

Or,
$$E(R_m) = 0.08 + (0.16 - 0.08)/1.15 = 0.1496$$

The new β is 1, and since the β of the market is also 1, this implies that

$$E(R) = E(R_m) = 0.1496$$

Thus
$$P_0 = 3/(0.1496 - 0.1) = \$60.53 \heartsuit$$

4.6. Eastern Oil stock currently sells at \$120 a share. The stockholders expect to get a dividend of \$6 next year, and they expect that the dividend will grow at the rate of 5% per annum. The expected return on the market is 12% and the riskless rate is 6%. This morning Eastern announced that it has won the multimillion dollar navy contract, and in response to the news, the stock jumped to \$125 a share. Find the beta of the stock before and after the announcement.

Using Gordon's growth model, $P_0 = \frac{D_1}{R - g}$, we get $R = D_1/P_0 + g$, which is also the expected return on the stock, $E(R)$. But by CAPM,

$$E(R_i) = r + \beta_i [E(R_m) - r]$$

we get
$$\beta = \frac{E(R_i) - r}{E(R_m) - r}$$

Thus
$$\beta = \frac{D_1/P_0 + g - r}{E(R_m) - r} = \frac{6/120 + 0.05 - 0.06}{0.12 - 0.06} = 0.667, \text{ before. } \heartsuit$$

And
$$\beta = \frac{6/125 + 0.05 - 0.06}{0.12 - 0.06} = 0.633, \text{ after. } \heartsuit$$

4.7. Jupiter Gas Company is planning to acquire Saturn Water Company. The additional pre-tax income from the acquisition will be \$100,000 in the first year, but it will increase by 2% in future years. Because of diversification, the beta of Jupiter will decrease from 1.00 to 0.9. Currently the return on the market is 12% and the riskless rate is 6%. What is the maximum price that Jupiter should pay for Saturn? The tax rate of Jupiter is 35%. The new beta for Jupiter is 0.9. Using CAPM, its expected return, and hence the cost of capital will be

$$E(R) = 0.06 + 0.9(0.12 - 0.06) = 0.114$$

$$\text{After tax income} = 100,000 (1 - 0.35) = \$65,000.$$

The total value of a firm is the present value of its future earnings, properly discounted. Thus, the value added to Jupiter due to the acquisition of Saturn is the present value of future after-tax earnings of Saturn, discounted at a rate equal to the cost of capital of Jupiter, and summed up to infinity. Thus

$$PV = \frac{65,000}{1.114} + \frac{65,000 (1.02)}{1.114^2} + \frac{65,000 (1.02)^2}{1.114^3} \dots \infty = \$691,489$$

Jupiter should pay at most \$691,489 for Saturn. \heartsuit

4.8. Hamlin Dairies stock has a beta of 1.33. It has just paid its annual dividend of \$1.20, and it sells for \$30 a share. Shareholders believe that Hamlin is growing at the rate of 7% annually and will maintain a constant dividend payout ratio. Due to the unexpected death of the chairperson, Hannibal Hamlin, the company is facing an uncertain future, and the price per share dropped to \$25. There is no other change in the company (dividends, growth, sales, etc.) or in the market. The riskless rate is 6%. In light of the greater risk of the company, find its new beta.

If the current dividend is \$1.20, next year it will be $1.20(1.07) = \$1.284$. Apply Gordon's growth model, (3.6), to find the required rate of return for the stockholders. Before Hamlin's death, it is

$$R = D_1/P_0 + g = 1.20(1.07)/30 + 0.07 = 0.1128 \text{ (before)}$$

After Hamlin's death, the stock price drops suddenly, but the growth potential and the current dividend remains intact. Thus the required rate of return after the death is

$$R = 1.20(1.07)/25 + 0.07 = 0.12136 \text{ (after)}$$

The expected and the required rate of return for the stock are the same, meaning $R = E(R)$. We can use these numbers in CAPM, (4.7), to get two equations:

$$\text{Before,} \quad 0.1128 = 0.06 + 1.33 [E(R_m) - 0.06]$$

$$\text{After,} \quad 0.12136 = 0.06 + \beta[E(R_m) - 0.06]$$

Put $\beta = x$ and $E(R_m) = y$ temporarily. Copy and paste the following instruction at [WolframAlpha](#) to solve the two equations simultaneously.

$$.1128 = .06 + 1.33 * (y - .06) , .12136 = .06 + x * (y - .06)$$

The approximate solution is $x \approx 1.54562$ and $y \approx 0.0996992$. Solving for beta, we get,

$$\beta = 1.55 \heartsuit$$

The new β , 1.55, is higher than the previous β , 1.33, because of the uncertainty created by the death of the chairperson. Greater uncertainty also means greater risk.

4.9. Epperly Fund invests in S&P500 companies and thereby simulates a market portfolio. The expected return of Epperly is 13.5%, with a standard deviation of 10%. Suppose you are able to borrow \$10,000 at the riskless rate of 9%, and you already have \$10,000 of your own money. If you invest this \$20,000 in Epperly Fund, what is the probability that you will have a return greater than 25% on your own money?

The β of the market is 1, by definition. Epperly Fund mimics the market and therefore, its β is also 1. When you borrow money to buy securities, the amount of borrowing is equivalent to a negative cash position in your account. The β of cash is zero, because the value of cash does not change due to fluctuations in the stock market. The total value of the portfolio you own is \$10,000, which equals your investment. Its composition is as follows:

	Value	β	Weight
Epperly Fund	\$20,000	1	2
Cash	-10,000	0	-1
Portfolio	10,000	2	1

To find the β of the portfolio, use

$$\beta_p = w_1\beta_1 + w_2\beta_2 = 2(1) + (-1)(0) = 2$$

This is highlighted in the previous table. With the help of CAPM, find

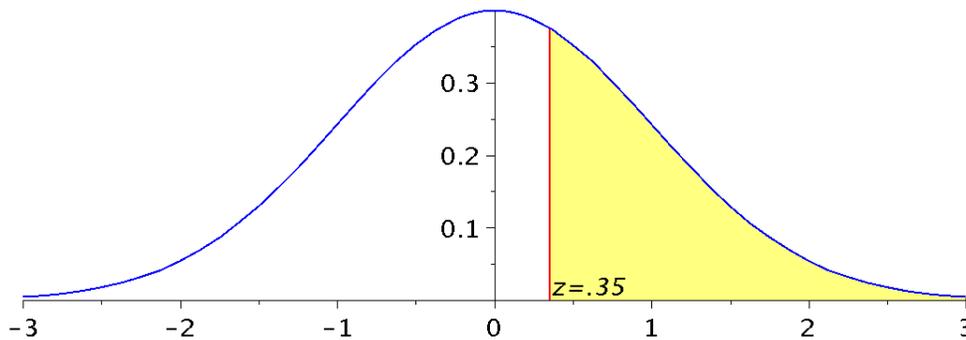
$$E(R_p) = .09 + 2(.135 - .09) = .18$$

The portfolio is consisting of two items with weights 2 and -1 . The σ of the components is, 10% for Epperly Fund and zero for cash. Using equation (6.4) for the σ of a two-security portfolio, we have

$$\sigma_p = \sqrt{(2)^2(.1)^2 + (-1)^2(0) + 2(2)(-1)(.1)(0)r_{12}}$$

Solving the above equation, we get $\sigma_p = .2$. We note that the expected return of the portfolio is 18% with a standard deviation of 20%. The required return is *more than 25%*. The probability of getting that return is *less than 50%*. To calculate it, first find

$$z = (25 - 18)/20 = 0.35$$



Draw a normal probability distribution curve, with $z = 0$ at the center and $z = .35$ to the right of center. The probability of getting a return of greater than 25% is equal to the shaded area to the right of $z = .35$. From the table, we get its value as,

$$P(R > 0.25) = 0.5 - .1368 = .3632 = 36.32\% \quad \heartsuit$$

4.10. Markham Co paid a dividend of \$3.00 yesterday, but these dividends are expected to grow at the rate of 5% in the long run. The beta of Markham is 0.95, the expected return on the market is 15%, and the riskless rate is 10% at present. Find the price of one share of Markham stock.

Using the CAPM, we have, $E(R) = 0.10 + 0.95(0.15 - 0.1) = 0.1475$

Using Gordon's growth model, we get the price of a share as

$$P_0 = 3(1.05)/(0.1475 - 0.05) = \$32.31 \quad \heartsuit$$

4.11. You have developed the following information about two mutual funds:

Name of fund	Beta	Expected return
Dione Market Fund	1.00	14%
Rhea Energy Fund	0.80	13%

You have \$5,000 to invest and you put \$3,000 in Dione and \$1,000 each in Rhea and riskless bonds. Find the beta and expected return of your portfolio.

Let us first find the riskless rate. Dione has β of 1, the same as that of the market. Thus the expected return of the market is also 0.14. Using CAPM, and using the information about Rhea,

$$0.13 = r + 0.8(0.14 - r)$$

which gives the riskless rate, $r = .09$. The weights of securities are

$$w_1 = 0.6, \quad w_2 = 0.2, \quad \text{and} \quad w_3 = 0.2.$$

The beta of the portfolio is just the weighted average of the betas of the individual securities. That is,

$$\beta_p = 0.6(1.00) + 0.2(0.8) + 0.2(0) = 0.76 \heartsuit$$

Similarly, the expected return on the portfolio is given by

$$E(R_p) = 0.6(0.14) + 0.2(0.13) + 0.2(0.09) = 0.128 \heartsuit$$

4.12. Pindar Corporation stock is selling for \$80 a share and its dividend next year is expected to be \$2. S&P500 index is 1437 at present, and it is expected to go up to 1550 after one year. The average dividend yield for the S&P500 is 1.52%, and the riskless rate is 5.14%. If the beta of Pindar is 1.14, find the expected price of one share of Pindar after one year.

Using the information about the market, find the expected percentage return on the market as the sum of the dividend yield of the market and its price appreciation,

$$E(R_m) = 0.0152 + (1550 - 1437)/1437 = .09384$$

Next, find the expected return of Pindar using CAPM,

$$E(R_j) = 0.0514 + 1.14(.09384 - 0.0514) = .09978$$

If x is the expected price of the stock next year, then the stock return, $(x - 80 + 2)/80 = .09978$. This gives $x = \$85.98 \heartsuit$

4.13. A portfolio is formed as follows:

Stock	Amount invested	β	$\sigma(R)$
Childs Corporation	\$12,000	1.25	25%
Jermyn Company	\$13,000	1.20	22%

The riskless rate is 7%, and the expected return on the market is 14%. The covariance between the two stocks is 0.0385. Find the expected return, and the standard deviation of returns of the portfolio, in dollars, and as a percentage.

It is easy to see that the weights are $w_1 = 12/25 = 0.48$, and $w_2 = 0.52$

The expected returns are $E(R_1) = 0.07 + 1.25(0.14 - 0.07) = 0.1575$

and $E(R_2) = 0.07 + 1.2(0.07) = 0.1540$

The portfolio return is $E(R_p) = 0.48(0.1575) + 0.52(0.154) = 15.568\% \heartsuit$
 $= 0.15568(25,000) = \$3892 \heartsuit$

The correlation coefficient $= \frac{0.0385}{(0.25)(0.22)} = 0.7$

The portfolio sigma, $\sigma(R_p) = \sqrt{0.48^2 0.25^2 + 0.52^2 0.22^2 + 2(0.48)(0.52)(0.25)(0.22)(0.7)}$
 $= 21.61\% \heartsuit$

And in dollars, $\sigma(R_p) = 0.2161(25,000) = \$5403 \heartsuit$

4.14. The β of Blakely Company is 1.2. Blakely is planning to acquire Waymart Corporation which will result in the combined company to have a β of 1.3. The riskless rate is 6%, and the expected return on the market is 12%. Waymart Corporation is expected to have first year earnings after taxes of \$40,000, and these earnings are expected to increase by 3% per annum in future. How much should Blakely pay for Waymart?

Risk adjusted discount rate $= 0.06 + 1.3(0.12 - 0.06) = 0.138$

$$\text{PV of earnings} = \frac{40,000}{1.138} + \frac{40,000(1.03)}{1.138^2} + \frac{40,000(1.03)^2}{1.138^3} + \dots \infty = \frac{\frac{40,000}{1.138}}{1 - \frac{1.03}{1.138}} = \$370,370$$

The value of Waymart is thus \$370,370 \heartsuit

4.15. McNamara Fund has expected return of 10.5%, standard deviation of 17.5%, and beta of 0.8. Schlesinger Fund has expected return of 12.5%, standard deviation of 21% and beta of 1.1. The two mutual funds have correlation coefficient of 0.7. Find the expected return and standard deviation of the market. What is the riskless rate of return?

First we use the CAPM, and put the numbers for the two funds, which gives us

$$\text{McNamara:} \quad 0.105 = r + 0.8 [E(R_m) - r]$$

$$\text{Schlesinger:} \quad 0.125 = r + 1.1 [E(R_m) - r]$$

To solve the equations, put $r = x$ and $E(R_m) = y$ temporarily. Then copy and paste the following instruction at [WolframAlpha](https://www.wolframalpha.com).

$$.105=x+.8*(y-x) , .125=x+1.1*(y-x)$$

Solving the two equations, we get $E(R_m) = 0.1183$, and $r = 0.05167$ ♥

Next we construct the market portfolio out of these two funds. The weights are w_1 and w_2 , and they are combined to get $\beta = 1$ for the market portfolio. Thus, we have

$$\begin{aligned}w_1 + w_2 &= 1 \\0.8 w_1 + 1.1 w_2 &= 1\end{aligned}$$

Solving these two equations we get, $w_1 = 1/3$ and $w_2 = 2/3$. The same result can be obtained by combining the expected returns of the two funds to get the expected return of the market. Now we can find the sigma of the market as follows:

$$\sigma(R_m) = \sqrt{(1/3)^2 0.175^2 + (2/3)^2 0.2^2 + 2(1/3)(2/3)(0.175)(0.21)(0.7)} = 0.1856 \text{ ♥}$$

4.16. Armstrong Corporation \$6 preferred stock sells for \$50 a share. The beta of this stock is 1.25. The current riskless rate is 8%. Just yesterday, Louis Armstrong, the founder and CEO, died and the stock dropped to \$47 a share in response to the news. Find the new beta of Armstrong preferred.

A preferred stock has fixed dividends, that is, there is no expectation of growth. This means $g = 0$ in Gordon's growth model, $P_0 = D_1/(R - g)$, which becomes $P_0 = D_1/R$. Rewrite it as $R = D_1/P_0$. This implies that the current return of the stock is $6/50 = .12$.

This is quite reasonable. If you buy a stock for \$50 a share and it pays a dividend of \$6 annually, without any growth opportunity, the return is indeed 12%.

Using CAPM,

$$.12 = .08 + 1.25[E(R_m) - .08]$$

Solve this equation to get $E(R_m) = (.12 - .08)/1.25 + .08 = .112$

This is quite reasonable, because the stock, with its $\beta_1 = 1.25$, has a return of .12; and the market with its $\beta = 1$, should have a lower expected return, perhaps around 11%.

Let $\beta_1 = 1.25$, beta of the stock before Armstrong's death and $\beta_2 =$ beta of the stock after his death. Now assume that Armstrong is just an insignificant player in the stock market, and the market will ignore his demise. The expected return on the market will remain at .112 and the risk-free rate at .08. The new return on the stock is $6/47$. The CAPM gives us

$$6/47 = .08 + \beta_2[.112 - .08]$$

This gives $\beta_2 = (6/47 - .08)/(.112 - .08) = 1.49$ ♥

The answer is quite reasonable, because Louis Armstrong was a very important individual at Armstrong Company. His departure has introduced a substantial measure of uncertainty, or risk, in the company, thereby increasing its β from 1.25 to 1.49.

Problems

4.17. The Washington Corp stock has a β of 1.15 and it will pay a dividend of \$2.50 next year. The expected rate of return of the market is 17% and the current riskless rate is 9%. The expected rate of growth of Washington is 4%. Find the value of its common stock.

\$17.61 per share ♥

4.18. Molopo Company has $\beta = 1.2$, whereas the return on the market is expected to be 12%, with a standard deviation of 8%. The riskless rate is 6% at present. The stock of Molopo is selling at \$100 a share, but it does not pay any dividends. Find the probability that it will be selling for more than \$120 by next year. Assume that the entire change in the stock price is due to the change in the market.

23.94% ♥

4.19. Cheever Corp stock is selling at \$40 a share. Its dividend next year will be \$2 a share and its beta is 1.25. Crane Company has the same growth rate as Cheever. The current stock price of Crane is \$55 a share, and its dividend this year is \$3. The riskless rate is 8% and the expected return on the market is 16%. Find the beta of Crane stock.

1.3955 ♥

4.20. Kingston Corporation has $\beta = 1.2$. It is interested in buying Plains Corporation which also has $\beta = 1.2$. Kingston believes that after the acquisition, its β will be 1.1. The expected after-tax earnings from Plains will be \$50,000 for the first year, but this figure is expected to increase by 3% per year in future. The expected return on the market is 12%, and the riskless rate is 6%. Find the amount that Kingston should spend on this acquisition.

\$520,833 ♥

4.21. Toledo Corporation estimates its β as 1.3, whereas the risk-free rate is 5% at present. The expected return on the market is 11%, with a standard deviation of 7%. Assume that the variation in the Toledo stock price is entirely due to the fluctuations of the market. If you invest \$10,000 in Toledo stock now, what is the probability that the value of your investment will be more than \$12,000 by next year?

21.45% ♥

4.22. Palmer Company stock has paid a dividend of \$1.25 this year, which is in line with its long-term growth rate of 5%. Its current β is 1.2 and the expected return of the market is 12%. Today, after the company won the multimillion-dollar contract from the navy, the stock jumped 3%, to \$15.45 a share, in response to the good news. Find the risk-free rate and the new β of the stock.

$r = 3.25\%$, new $\beta = 1.171$ ♥

4.23. Johnson Corporation preferred stock sells for \$37 a share and pays an annual dividend of \$4. The β of this stock is 1.3. The current riskless rate is 3%. The common stock of Johnson was upgraded by the analysts from 'hold' to 'buy' today. In response to the news, the preferred stock jumped in price by \$1. Find the new β of Johnson preferred.

1.253 ♥

Key Terms

beta, 116, 117, 118, 119, 120,
121, 123, 124, 125, 126,
127, 128, 129, 130, 131

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