

3. BASICS OF PORTFOLIO THEORY

Goals: After reading this chapter, you will

1. Understand the basic reason for constructing a portfolio.
2. Calculate the risk and return characteristics of a portfolio.
3. Develop the basic formulas for two-, three-, and n -security portfolios.

3.1 [Video 06A](#), Return of an Investment

When people buy the stock of a corporation, they expect to make money in two ways: from the appreciation in the value of the stock, and from the dividends. We define the total return R of a common stock, for a certain holding period, as

$$R = \frac{P_1 - P_0 + D_1}{P_0} \quad (3.1)$$

Here P_0 is the price of the stock at the beginning of the period, P_1 is the price at the end of that period, and D_1 is the dividend paid by the stock at the end of the period. This return is the historical return, or *ex post* return.

If the returns over n periods for a security are R_1, R_2, R_3, \dots , then we can find the [arithmetic mean](#) of the returns by adding the returns for each of the periods, and dividing the sum by the number of periods. That is,

$$AM(R) = \frac{1}{n} \sum_{i=1}^n R_i \quad (3.2)$$

For instance, if a stock gives a return of 5% in the first year and 15% in the second year, the arithmetic mean return is 10%. Does it mean that the total return for the two-year period is 20%? Not really, because if the initial stock price is \$100, then its value with the dividends reinvested, will be $100 \cdot (1.05) \cdot (1.15) = \120.75 at the end of two years. On the other hand, the 20% return over the two-year period gives a final value of \$120.00. This implies that the arithmetic average is not a reliable measure of average rate of return for an investment.

Since the returns have a multiplicative effect on the final value of an investment, the proper mean return should be the [geometric mean](#). We can define the geometric mean as

$$GM(R) = [(1 + R_1)(1 + R_2)(1 + R_3)\dots]^{1/n} - 1 \quad (3.3)$$

For financial assets, the $GM(R)$ is the more meaningful rate of return, because this gives the compound rate of growth. In the previous example, we find the geometric mean to be $\sqrt{1.05 \cdot 1.15} - 1 = .098863 = 9.8863\%$. Performing the calculation to find the value of the investment after two years, we get $100(1.098863)^2 = \$120.75$.

If we have to find the rate of growth of a mutual fund over a period of several years, we should calculate the geometric mean of its annual returns. The newspaper advertisements of various mutual funds also provide this number. Further, $GM(R)$ is always less than $AM(R)$, which makes the advertised return to be more conservative.

It is well known that the actual, or realized, returns from a stock are quite random. The returns can also be negative. Suppose we observe the price of a non-dividend paying stock for a number of days, as $P_1, P_2, P_3 \dots$ then the quantity

$$R = \ln(P_{i+1}/P_i) \quad (3.4)$$

is seen to be approximately normally distributed. Here P_i is the price of the stock on a given day, P_{i+1} is the price on the next trading day, and R is the [continuously compounded](#) rate of return per day on the stock investment. Given the nature of equation (2.4) we may also say that the stock prices have a lognormal distribution.

3.2 Risk of an Investment

The concept of risk pervades throughout finance. This is especially true of portfolio theory. Every investment carries a certain amount of risk. When we buy a stock we expect to make, say 12% on it in the next year. However, this 12% return is not guaranteed by any means. We may end up making 20% on the investment, or we may even lose 20%. There are also risk-free securities available to the investors. One such example of a risk-free investment is a Treasury bond.

The two main characteristics of a portfolio are its *risk* and *return*. Since the return of any investment is uncertain, one should look at the *expected* return of a portfolio. The expected return of a stock is more difficult to find. However, there are various ways to estimate the future return of a stock. We shall consider them in section 7.

Although it is quite difficult to quantify risk, one useful measure of risk is the standard deviation of the returns, designated by σ .

3.3 Portfolio Formation

A portfolio is collection of projects, or securities, or investments, held together as a bundle. For example you may buy 100 shares of Boeing, 200 hundred shares of Microsoft, and 5 PP&L bonds in an account. This is your portfolio of investments. Individual investors have a portfolio of investments that may include real estate (the family residence), some stocks, bonds, or shares of mutual funds, and possibly some money accumulated in a pension plan. A portfolio may also include less tangible items as your professional education, or even a license to practice law. The total value of your portfolio may fluctuate with time.

As an investor, you may open an account at a brokerage house by depositing some cash. You may apply for [margin](#) privileges, meaning that you may buy securities with borrowed

money. At present, the margin rate is 50%. This implies you must put at least 50% of your own money to buy a certain number of shares of stock. The brokerage house may have its own policies regarding the margin account. If the value of the stock drops, you must deposit more funds to *maintain* your margin. You may receive a margin call, if your part of the account value drops below a certain level. Currently, the regulations require that you maintain the margin at 35%, that is, your equity in the account should be at least 35%

Corporations also have a portfolio of different projects. They carefully select profitable projects and invest in them. The banks loan money to individuals and corporations. They have a loan portfolio, and they try to monitor the quality of their portfolios. The quality of a loan portfolio can deteriorate if too many loans are non-performing.

Why do people, or corporations, form a portfolio? The simple answer is diversification. You do not want to risk everything on one endeavor. It is a good idea to diversify your risk, and if some of the investments do not pan out, the others will keep the value of the portfolio intact.

The two main features of a portfolio are its *risk* and *expected return*. In 1952, [Harry Markowitz](#) first developed the ideas of portfolio theory based upon statistical reasoning. He showed that an investor could reduce the risk for a given return by putting together unrelated or negatively correlated securities in a portfolio. Section 5.4 gives a summary of Markowitz' analysis.

3.4 Risk and Return of a Portfolio

We start by looking at the simplest portfolio, the one that has only two securities in it. For a two-security portfolio, the weights of the two securities w_1 and w_2 must add up to one. This means

$$w_1 + w_2 = 1 \quad (3.5)$$

The expected return of the portfolio is simply the weighted average of the expected returns of the individual securities in the portfolio. For a two-security portfolio, this comes out to be

$$E(R_p) = w_1E(R_1) + w_2E(R_2) \quad (3.6)$$

Here $E(R_p)$ is the expected return of the portfolio, $E(R_1)$ and $E(R_2)$ are the expected returns of the individual securities.

Combining the risk of the two securities, σ_1 and σ_2 , we get the composite risk of the portfolio $\sigma(R_p)$ to be

$$\sigma(R_p) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2r_{12}} \quad (3.7)$$

Here r_{12} is the correlation coefficient between the securities. The correlation coefficient effectively measures the overlap, or interaction, between the two securities. If the securities are highly correlated, they tend to mimic each other in their performance. The value of the correlation coefficient lies somewhere between +1 and -1. For any two completely unrelated securities, the correlation coefficient between them is zero, $r_{ij} = 0$. For perfectly positively correlated securities, $r_{ij} = 1$, and for perfectly negatively correlated ones, $r_{ij} = -1$. In real life, most of the securities are partially positively correlated with one another.

By definition, the covariance between the returns of the securities i and j is equal to the product of the correlation coefficient between these securities and the standard deviations of the returns of these two securities, as seen in previous section

$$\text{cov}(i,j) = r_{ij}\sigma_i\sigma_j \quad (2.5)$$

Let us extend the previous results by constructing a portfolio with three assets. The weights should add up to one,

$$w_1 + w_2 + w_3 = 1$$

The expected return of the portfolio is still the weighted average of the expected returns of the individual securities. We may express it as

$$E(R_p) = w_1E(R_1) + w_2E(R_2) + w_3E(R_3)$$

Likewise, we may construct the expression for the σ of a three-security portfolio. First, we have three terms containing the risk of the individual securities, and then three more terms due to the interaction between the securities:

$$\sigma(R_p) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_1\sigma_2r_{12} + 2w_1w_3\sigma_1\sigma_3r_{13} + 2w_2w_3\sigma_2\sigma_3r_{23}}$$

In the above equation r_{12} is the correlation coefficient between the first and the second security, r_{13} between the first and the third one, and r_{23} between the second and the third one.

For a portfolio with n securities, we may generalize the above equations as follows. First, the weights of all securities must add up to 1. We write this as

$$\sum_{i=1}^n w_i = 1 \quad (3.8)$$

Second, the expected return is still the weighted average of the returns of all the securities expressed as

$$E(R_p) = \sum_{i=1}^n w_i E(R_i) \quad (3.9)$$

The standard deviation of the returns of the portfolio is a measure of the uncertainty in the expected returns. This uncertainty will depend upon the uncertainty in the performance of component securities, the weights of these securities, and how are these securities correlated. Negatively correlated securities will tend to cancel out the uncertainty in the portfolio. Even positively correlated securities (except when they are 100% positively correlated) will tend to reduce uncertainty in its portfolio. In general, we may express it as

$$\sigma(R_p) = \left[\sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{cov}(i,j) \right]^{1/2} \quad (3.10)$$

Suppose we know the initial investments and expected returns in *dollars*, rather than percentages, then we may write the above formulas as

$$E(R_p) = \sum_{i=1}^n E(R_i) \quad (3.11)$$

$$\sigma(R_p) = \left[\sum_{i=1}^n \sum_{j=1}^n \text{cov}(R_i, R_j) \right]^{1/2} \quad (3.12)$$

For example, for a two-security portfolio (3.11) and (3.12) become

$$E(R_p) = E(R_1) + E(R_2) \quad (3.13)$$

$$\sigma(R_p) = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2r_{12}} \quad (3.14)$$

The following examples will help us understand the use of these formulas.

Examples

3.1. Suppose you bought Amherst Company stock at a price of \$77 a share and sold it one month later at \$82 a share. You also received a dividend of \$1.00 at the end of the month. Find your annual rate of return, assuming monthly compounding.

$$R(\text{monthly}) = (82 - 77 + 1)/77 = 6/77 = 7.79\%$$

$$R(\text{annual}) = (1.0779)^{12} - 1 = 146\% \quad \heartsuit$$

3.2. The following table gives the price of Andover Company stock, along with the annual dividend, paid at the end of each year. Find its annual return for the 5-year period.

Year	Initial price	Final price	Dividend
2011	\$40.00	\$41.00	\$1.50
2012	41.00	44.00	1.50
2013	44.00	55.00	1.50
2014	55.00	59.00	1.75
2015	59.00	70.00	1.75

To find the total return for a given year, we add the price appreciation and the cash dividend, and then divide the sum by the initial price of the stock. For the first year, it is $(1 + 1.50)/40$. The compound rate of return for five years is thus

$$\text{Total } R = (1 + 2.5/40)(1 + 4.5/41)(1 + 12.5/44)(1 + 5.75/55)(1 + 12.75/59) = 2.033788$$

We find the geometric mean of the returns by taking the fifth root of the total return.

$$\text{GM}(R) = (2.033788)^{1/5} - 1 = 15.26\% \heartsuit$$

3.3. You opened a margin account by depositing \$10,000. Then you bought 1200 shares of Arlington stock at \$15 per share. At the end of one year, you received a cash dividend of \$1.20 per share, and then sold the stock at \$17 a share. The broker charged 6% interest on the borrowed funds. Find your rate of return.

Your original investment is \$10,000. You want to buy $1200 \times 15 = \$18,000$ worth of stock. You can do so by borrowing \$8,000 from the broker. At the end of the year, you receive $1200 \times 1.20 = \$1440$ in dividends. When you sell the stock, you will get $1200 \times 17 = \$20,400$. Out of this, you must pay the margin loan with interest, which amounts to $8000(1.06) = \$8480$. Thus your total payoff is $1440 + 20,400 - 8480 = \$13,360$. Your profit is \$3360. The rate of return on your investment is $3360/10,000 = 33.60\%$. \heartsuit

3.4. [Video 06.01](#) Cooper Corporation has the opportunity to invest in only two of the following three projects:

	Project A	Project B	Project C
Expected <i>NPV</i>	\$10,000	\$11,000	\$9,000
Standard deviation	\$2,000	\$1,900	\$1,500
Corr. coeff. between	(A,B) = .4	(A,C) = .5	(B,C) = .8

Which two projects should the company select, if it wants to maximize the ratio between expected *NPV* and the standard deviation?

For a portfolio with two projects A and B, the sigma is given by (3.14),

$$\sigma(R_p) = \sqrt{\sigma_A^2 + \sigma_B^2 + 2\sigma_A\sigma_B r_{AB}}$$

For A + B, *NPV* = \$21,000

$$\sigma = \sqrt{2000^2 + 1900^2 + 2(0.4)(2000)(1900)} = \$3263$$

$$\text{NPV}/\sigma = 21,000/3263 = 6.4349$$

For A + C, *NPV* = \$19,000

$$\sigma = \sqrt{2000^2 + 1500^2 + 2(0.5)(2000)(1500)} = \$3041$$

$$NPV/\sigma = 19,000/3041 = 6.2472$$

For B + C, $NPV = \$20,000$

$$\sigma = \sqrt{1900^2 + 1500^2 + 2(0.8)(1900)(1500)} = \$3228$$

$$NPV/\sigma = 20,000/3228 = 6.1958$$

Take A and B. ♥

3.5. Suppose you want to invest \$40,000 in Barnstable stock, whose expected return is 15% with standard deviation 20%, and \$10,000 in Belmont stock whose expected return is 18% with standard deviation 21%. The correlation coefficient between the securities is 0.75. Find the expected total value of your portfolio after one year, and its standard deviation in dollars.

First, we find the weights of the two securities in the portfolio. It works out to be

$$w_1 = 0.8, w_2 = 0.2$$

The expected return of the portfolio comes out as

$$E(R_p) = 0.8(0.15) + 0.2(0.18) = 0.156$$

The standard deviation of the return of portfolio is

$$\sigma(R_p) = \sqrt{(.8)^2(.2)^2 + (.2)^2(.21)^2 + 2(.8)(.2)(.21)(.2)(.75)} = .1935$$

To find the expected value of the portfolio, and its standard deviation, in dollars, we calculate

$$E(V) = 50,000 (1.156) = \$57,800 \text{ ♥}$$

and

$$\sigma(V) = 50,000 (0.1935) = \$9,675 \text{ ♥}$$

3.6. You want to make a portfolio of Beverly Company and Boston Company common stocks. Beverly has an expected return of 12% and sigma .25, the expected return of Boston is 15% and its sigma 0.30. The coefficient of correlation between the two companies is 0.25. The return of the portfolio is 13%. What is the sigma of the portfolio?

We write the expected return of the portfolio as

$$E(R_p) = w_1R_1 + w_2R_2 = w_1R_1 + (1 - w_1)R_2$$

Substituting the numbers, we have

$$.13 = w_1(.12) + (1 - w_1)(.15)$$

Solving for w_1 , we get $w_1 = 2/3$. If $w_1 = 2/3$, then $w_2 = 1/3$. Using (3.7), we get

$$\sigma(R_p) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2r_{12}} = 0.2147 \heartsuit$$

3.7. Video 06.06 Elizabeth Corporation is starting two new projects. Project A requires an investment of \$5,000, has expected return of 16% with standard deviation 14%. Project B has initial investment of \$15,000, expected return of 15% with standard deviation 10%. The correlation coefficient between the projects is 0.75. Find the expected return, in dollars, of the portfolio of these two projects. What is the probability that this return is less than \$4,000?

The total value of the portfolio is \$20,000 and the weights are 0.25 and 0.75. We calculate the expected return of the portfolio as

$$E(R_p) = 0.25(0.16) + 0.75(0.15) = 0.1525$$

And in dollars, $E(R_p) = 0.1525(20,000) = \$3,050$

To find the sigma of the portfolio, use

$$\text{Sigma of portfolio, } \sigma(R_p) = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2r_{12}} \quad (3.7)$$

$$\sigma(R_p) = \sqrt{.25^2(.14^2) + .75^2(.1^2) + 2(.25)(.75)(.14)(.1)(.75)} = 0.1038629$$

If the return is less than \$4,000, it is less than $4,000/20,000 = 0.2 = 20\%$.

Draw a normal probability distribution curve, with $\mu = .1525$, and $\sigma = .1039$. The point for 20% return will be somewhat to the right of center, and about one-half standard deviation off.

Thus $z = (0.2 - 0.1525)/0.1038629 = 0.4573$.

If the return is less than 20%, it will be the area under the curve to the left of this 20%-point. This area is more than half the area under the curve. Therefore, the answer should be more than 50%. Checking the numbers corresponding to 0.4573 in the tables,

$$P(R < \$4,000) = 0.5 + 0.1736 + .73(.1772 - .1736) = 0.6763 = 67.63\% \heartsuit$$

3.8. The following table gives the expected return of two stocks, A and B, under different states of the economy. The investment in Stock A is \$220,000 and that in Stock B \$280,000.

State of Economy	Probability	Return on A	Return on B
Poor	0.5	0.02	0.01
Average	0.3	0.10	0.11
Good	0.2	0.15	0.21

Calculate the expected return and standard deviation of the portfolio of these two stocks in dollars.

The total investment in the portfolio is \$500,000. The weights of the two stocks are: $w_1 = 0.44$, and $w_2 = 0.56$. Using various formulas, we find

$$\begin{aligned} \text{From (2.5), } E(R_1) &= 0.5(0.02) + 0.3(0.1) + 0.2(0.15) = 0.07 \\ E(R_2) &= 0.5(0.01) + 0.3(0.11) + 0.2(0.21) = 0.08 \end{aligned}$$

$$\text{From (2.12), } E(R_p) = 0.44(0.07) + 0.56(0.08) = 0.0756 \quad \heartsuit$$

$$\text{From (2.6), } \text{var}(R_1) = .5(.05)^2 + .3(.03)^2 + .2(.08)^2 = 0.0028$$

$$\text{var}(R_2) = .5(.07)^2 + .3(.03)^2 + .2(.13)^2 = 0.0061$$

$$\text{From (2.8), } \text{cov}(R_1, R_2) = .5(-.05)(-.07) + .3(.03)(.03) + .2(.08)(.13) = 0.0041$$

$$\begin{aligned} \text{From (2.13), } \text{var}(R_p) &= (.44)^2 (.0028) + (.56)^2 (.0061) + 2(.44)(.56)(.0041) = 0.0044755 \\ \sigma(R_p) &= 0.0669 \quad \heartsuit \end{aligned}$$

We can express the results in dollars as: $E(R_p) = 0.0756(500,000) = \$37,800$,

$$\sigma(R_p) = 0.0669(500,000) = \$33,450 \quad \heartsuit$$

3.9. You are going to make a portfolio out of these three securities:

Stock	Weight	E(R)	σ
Cranston	20%	15%	30%
Providence	30%	18%	40%
Warwick	50%	20%	50%

The correlation coefficient between any two securities is 0.5. Find the expected return and sigma of the portfolio. Show the individual stocks and the portfolio on a risk-return diagram.

$$E(R_p) = .2(.15) + .3(.18) + .5(.2) = .184 = 18.4\%$$

$$\sigma(R_p) = \sqrt{.2^2(.3)^2 + .3^2(.4)^2 + .5^2(.5)^2 + 2(.2)(.3)(.3)(.4)(.5) + 2(.2)(.5)(.3)(.5)(.5) + 2(.3)(.5)(.4)(.5)(.5)} = .3643$$

Plotting these values on a risk-return diagram, we have:

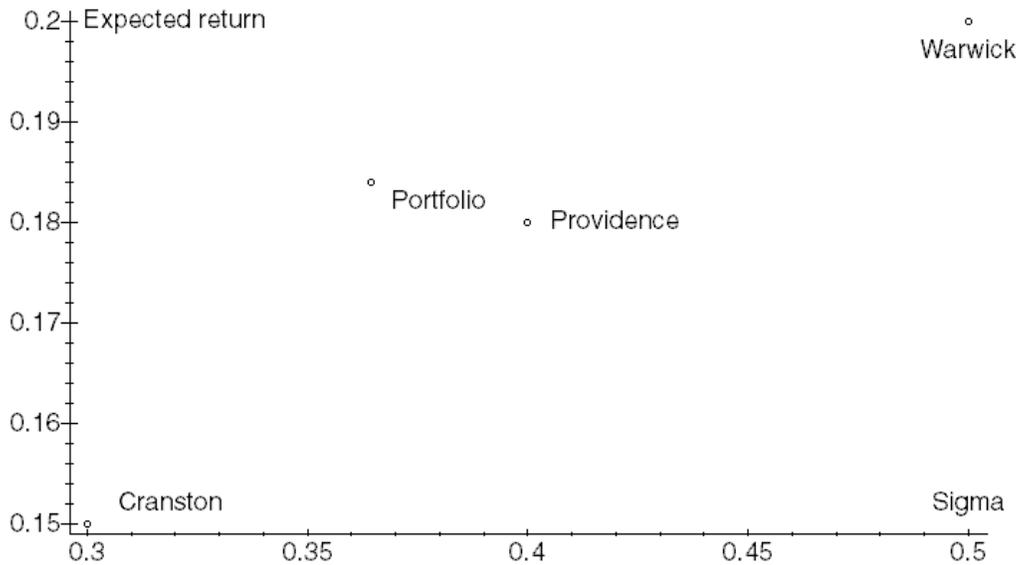


Fig. 3.1. The diagram shows the expected return and standard deviation of returns for three securities, and the expected return and standard deviation of their portfolio.

Problems

3.10. The following table shows the stock price for Bridgewater Corporation for the 2001-2005, along with the dividend paid at the end of each year. Calculate its arithmetic average, and geometric average rate of return for this period.

Year	Price at the start of the year	Price at the end of the year	Dividend paid at the end of the year
2011	\$22.00	\$24.00	\$1.00
2012	\$24.00	\$36.00	\$1.00
2013	\$36.00	\$30.00	\$1.00
2014	\$30.00	\$33.50	\$1.50
2015	\$33.50	\$40.00	\$1.50

AM = 18.89%, GM = 16.87% ♥

3.11. You opened a margin account by depositing \$25,000. Then you bought 1800 shares of Brockton Company stock at \$21 a share. The margin maintenance rate is 35%. Find the price of the stock when you will get a margin call. \$10.94 ♥

3.12. You have \$50,000 that you would like to invest in two companies, Brookline and Cambridge. Brookline has a return of 10% and standard deviation 45%, while Cambridge has return of 15% with a standard deviation of 55%. The correlation coefficient between them is .5. Your portfolio should have a return of 12%. Find the standard deviation of this portfolio's returns. $\sigma(R_p) = 42.51\%$ ♥

3.13. Burlington Corporation is considering the following three projects:

Project	E(R)	$\sigma(R)$	Correlation coefficients
A	\$4,000	\$2,100	(A,B) 0.8
B	5,000	2,500	(A,C) 0.7
C	6,000	2,800	(B,C) 0.6

Burlington can take any one, any two or all three projects. If the company wants to maximize the ratio $E(R)/\sigma(R)$, what is the best course of action?

Invest in B and C. Ratio = 2.3195 for B + C ♥

3.14. Chelsea Corporation wants to make a portfolio out of these three stocks:

	Stock A	Stock B	Stock C
Initial investment	\$235,000	\$455,000	\$310,000
Expected return	12%	11%	13%
Standard deviation	30%	35%	40%
Correlation coefficients	(A,B) = .4	(A,C) = .5	(B,C) = .6

Find the probability that the return on the portfolio is more than 15%. 45.76% ♥

3.15. Chicopee Corporation has the opportunity to invest in any of the following three projects:

	Project A	Project B	Project C
Initial investment	\$10,000	\$11,000	\$12,000
Standard deviation of returns	\$2,000	\$2,500	\$2,800
Correlation coefficients	(A,B) = .4	(A,C) = .5	(B,C) = .8

If the company can invest in one, two, or three projects, what should it do to maximize the ratio between initial investment and standard deviation of returns?

I_0/σ ratios: A = 5, B = 4.4, C = 4.28, (A + B) = 5.5630, (A + C) = 5.2680, (B + C) = 4.5735, (A + B + C) = 5.2916. Take (A + B). ♥

3.16. You have to make a risk-free portfolio from these two stocks: Gloucester Company with expected return 12% and standard deviation 20%; and Haverhill Company with expected return 14% and standard deviation 25%. The correlation coefficient of the stocks is -1 . Find the risk-free rate of return. 12.89% ♥

3.17. Suppose you have \$40,000 that you would like to invest equally in four securities A, B, C, and D. The expected returns from these securities are 10%, 11%, 12%, and 13%, respectively. The standard deviation of these returns is 12%, 14%, 16%, and 18%, respectively. The correlation coefficient between any two securities is 0.8. If the returns

are normally distributed, what is the probability that the portfolio will be worth more than \$50,000 after one year.

$$E(R_p) = 0.115, \sigma(R_p) = 0.1384, z = 0.9755, P(R > 0.25) = 16.43\% \heartsuit$$

3.18. Concord Corporation is considering the following three stocks for investment:

Stock	Investment	E(R)	$\sigma(R)$	Corr. coeff.
A	\$10,000	0.15	0.20	A, B = 0.3
B	\$20,000	0.14	0.18	A, C = 0.5
C	\$50,000	0.12	0.16	B, C = 0.7

The company may buy any one, any two or all three stocks. The company wants to maximize the ratio $E(R)/\sigma(R)$. What do you recommend?

$$\text{Take the stocks A and B together, } E(R)/\sigma(R) = 0.9322 \heartsuit$$

3.19. Suppose you have formed a portfolio with 200 shares of Danvers Corporation priced at \$20 a share, and 150 shares of Easton Corporation valued at \$40 each. The return on these stocks under different states of economy is as follows:

State of Economy	Probability	Return on Danvers	Return on Easton
Good	0.5	0.25	0.20
Average	0.3	0.10	0.10
Poor	0.2	-0.10	-0.05

Find the expected return and sigma of the portfolio.

$$.1260, .1109 \heartsuit$$

3.20. A portfolio is made of 400 shares of Dedham Corporation, selling at \$20 each, and 1700 shares of Everett Corporation that sell at \$10 each. The following table shows the expected return for these securities under different conditions.

State of Economy	Probability	Return of Dedham	Return of Everett
Good	25%	22%	25%
Fair	50%	15%	15%
Poor	25%	5%	0%

Find the expected return and standard deviation of return of the portfolio.

$$E(R_p) = 13.91\%, \sigma(R_p) = 8.008\% \heartsuit$$

3.21. Dartmouth Company has made a portfolio of three stocks as follows:

	Investment	E(R)	$\sigma(R)$	Correl. coeff.
Stock A	\$25,000	10%	20%	(A,B) = .3
Stock B	\$35,000	12%	25%	(A,C) = .4
Stock C	\$40,000	15%	30%	(B,C) = .5

If the returns are normally distributed, find the probability that the return of the portfolio is more than 15%. $P(R > .15) = 45.56\%$ ♥

Multiple-Choice Questions

1. Suppose you buy a stock at \$50 a share and sell it two months later at \$55 a share. Your annual rate of return is

- A. 5% B. 10%
C. 60% D. 77%

2. You buy a stock for \$60 and sell it for \$62 after 3 months. You also receive a \$1 dividend just before you sell the stock. Then your compound annual rate of return is

- A. 14.01% B. 15.76%
C. 20.81% D. 21.55%

3. If the price of a non-dividend paying stock triples in three years, then its annual geometric mean rate of return is

- A. 33% B. 44%
C. 50% D. 100%

4. There are two stocks with $\sigma_1 = 0.2$, and $\sigma_2 = 0.3$. The correlation coefficient between them is 0.8. Then the covariance between them is

- A. 0.075% B. 4.80%
C. 13.33% D. 20.83%

5. The covariance between two stocks is .045, and their variances are .04 and .09. The correlation coefficient between them is

- A. 0.000162 B. 0.0027
C. 0.10125 D. 0.75