5. CAPITAL BUDGETING UNDER UNCERTAINTY

Objectives: After reading this chapter, you should
1. Understand the basic ideas of discrete and continuous probability distributions.
2. Apply the concepts of probability to the problems of financial decision-making.
3. Be able to analyze problems involving inflation.
4. Understand the role of options in the capital budgeting decisions.

5.1 Probability and Capital Budgeting

We do not know the outcome of many future events with certainty. One way to handle the problem is to use a probabilistic model that would describe the situation. This is especially true of financial decisions where we do not know the future cash flows exactly. One way to overcome this uncertainty is to develop a subjective probability distribution about different possible outcomes. To find the expected value of the uncertain outcome, we first multiply the probability of various possible outcomes with the value of each outcome, and then sum them all. We express this in the form of an equation:

$$\mathrm{E}(V) = \sum_{i=1}^{n} P_i V_i \tag{1.6}$$

Here $\mathrm{E}(V)$ is the expected value of an uncertain outcome, P_i is the probability that the outcome V_i will occur, and n is the total number of possible outcomes.

If there are a large number of independent observations of a random event, then we may approximate the result by the normal probability distribution. The two parameters describing the distribution are the mean, or the expected value μ of a variable, and its standard deviation σ. A smooth bell-shaped curve will represent the probabilities. The area under the curve represents the cumulative probability of a certain outcome. These values are available in a table set up so that the total area under the curve is unity, and the area under half the curve is 0.5. The table provides the values for the area under the curve, measured from the center, up to a point that is a certain number z of standard deviations away from the mean value.

Examples

Video 05A **5.1.** Mr. Barkis is considering an investment in Murdstone Inc stock. He believes that the continuously compounded returns of the stock have a normal distribution, with mean of 15%, and standard deviation 30%. What is the probability that the continuously compounded return is more than 20%? What is the probability that his loss is greater than 10%?

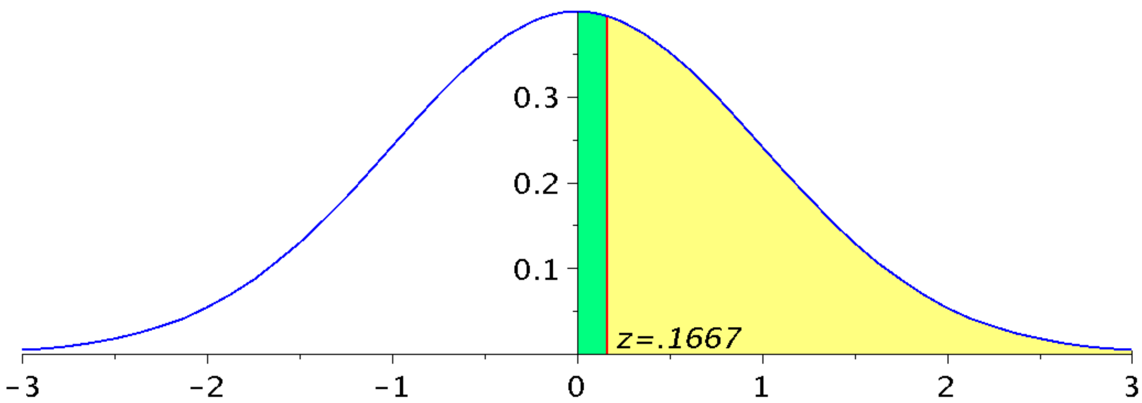
Using the concept of continuous compounding, we may relate the final stock price P_1 and the initial price P_0 by the expression $P_1 = P_0 e^{rT}$. Here T is a certain time period, say one year. We can find the value of the continuously compounded rate of return *r* from this expression as

$$r = \frac{\ln(P_1/P_0)}{T} \tag{5.1}$$

We define *z* as the number of standard deviations that a given value is away from the mean value. Here the mean value of returns is $\mu = 0.15$, the required return is $x = 0.2$, and the standard deviation is $\sigma = 0.3$. Since the required return is more than the expected return, it is unlikely that the stock will accomplish that. The resulting probability is less than 50%. Find *z* as

$$z = \frac{x-\mu}{\sigma} = \frac{0.2-0.15}{0.3} = \frac{0.05}{0.3} = 0.1667$$

Now draw the normal probability distribution curve with $z = 0$ in the center and $z = 0.1667$ a little right of the center. The area to right of $z = 0.1667$, shaded yellow, under the tail of the curve will represent the answer.



The probability table, in Chapter 16, shows the area under the curve from the mid point to a given point *z*. This is the green area in the above diagram. In our case, we have to find the probability of making more than 20% on our investment. This is equivalent to the yellow area on the right side of $z = .1667$. In the table, the area corresponding to $z = .16$ is .0636, and for $z = .17$, it is .0675. We have to find the area for $z = .1667$, which is 67% of the way between $z = .16$ and $z = .17$. We have to interpolate the numbers between .0636 and .0675. The total area under the curve is 1, half of it is .5, and so we have

$$P(r > 0.2) = .5 - [.0636 + .67(.0675 - .0636)] = 0.4338, \text{ or } 43.38\%. ♥$$

You can check the naswer at Excel by copying the following instruction.

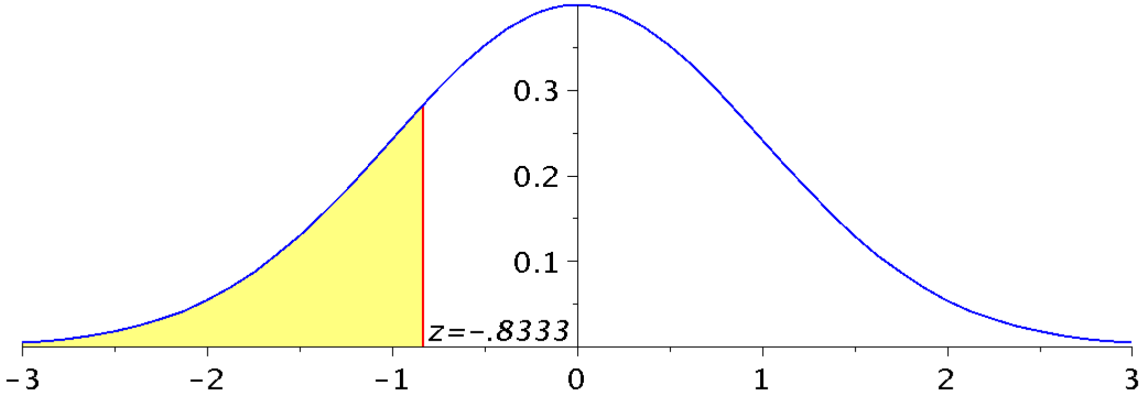
EXCEL =1-NORMDIST(.2,.15,.3,TRUE)

The answer is quite plausible. We expect to make 15%, and there is a good possibility that we may make more than 20% in view the standard deviation of 30%.

For a loss of 10%, the return is – 0.1. For the loss to be greater than 10%, the return must be less than –10%. We calculate *z* to be

$$z = \frac{x - \mu}{\sigma} = \frac{-0.1 - 0.15}{0.3} = -.8333$$

For a negative value of *z*, you just take the absolute value, because the probability table gives the area under the curve on either side of the mean value. This time we have to look for area on the left side of *z* = –.8333, under the tail of the curve, which corresponds to a return of less than –10%.



From the table, we have first get the area between $z = -.8333$ and the center of the curve. In the table, The area for $z = .83$ is .2967 and that for $z = .84$, it is .2995. Since we have to go 33% of the way from .83 to .84 to reach .8333, we intertoplate the table as follows.

$$P(r < -0.1) = .5 - [.2967 + .33(.2995 - .2967)] = .2024 \text{ or } 20.24\%.$$ ♥

This result seems reasonable because there is a fair chance that the stock can indeed go down by 10%.

	A	B
1	Expected return, μ	.15
2	Standard deviation, σ	.3
3	First required return, x	.2
4	z = (x – μ)/σ	=(B3-B1)/B2
5	Prob(R > .2)	=1-NORMDIST(B4,0,1,true)
6	Second required return, x	-.1
7	z = (x – μ)/σ	=(B6-B1)/B2
8	Prob(R < -.1)	=NORMDIST(B7,0,1,true)

In Excel, the function NORMDIST(x,mean,standard_dev,cumulative) serves the purpose of the probability table in Chapter 16. For instance, NORMDIST(x,.15,.3,true) will calculate the cumulative area from the left end of the curve up to point x. If you need the area to the right of point x, you enter 1 − NORMDIST(x,.15,.3,true). To do this problem, you can enter the following instructions.

EXCEL =NORMDIST(-.1,.15,.3,TRUE)

5.2. Black Ink is in financial distress. Its bonds have a 12% coupon rate and they pay the interest semiannually. There is a 70% chance that it will go bankrupt after 1 year, and it faces certain bankruptcy after 2 years. In case of bankruptcy, the company will pay interest due on the bonds, but will pay only 30% of the principal, at the end of that year. If your required rate of return is 12%, how much should you pay for a $1,000 Black Ink bond?

In this problem, we look at the probability of an outcome and multiply it with the dollar amount of that outcome, and then add all the products. There is a 70% chance that the company will go bankrupt after one year, and 30% that it will be bankrupt after two years. These are the only two possible outcomes because the company will not survive after two years.

If the company goes bankrupt after one year, the bond should pay two interest payments plus $300 at the end of one year. If it survives another year, it should pay four interest payments, plus $300 at the end of the second year. The semiannual discount rate is 6%. Multiplying the probability of an outcome with the present value of that outcome, and adding the results, we have

$$P_0 = 0.7\left(\sum_{i=1}^{2}\frac{60}{1.06^i}+\frac{300}{1.06^2}\right)+0.3\left(\sum_{i=1}^{4}\frac{60}{1.06^i}+\frac{300}{1.06^4}\right) \quad \textbf{(A)}$$

$$= 0.7\left(\frac{60(1-1.06^{-2})}{0.06}+\frac{300}{1.06^2}\right)+0.3\left(\frac{60(1-1.06^{-4})}{0.06}+\frac{300}{1.06^4}\right) = \$397.56 \ ♥$$

You may check the answer at WolframAlpha, by writing equation **(A)** as

WRA `.7*(sum(60/1.06^i,i=1..2)+300/1.06^2)+.3*(sum(60/1.06^i,i=1..4)+300/1.06^4)`

5.3. Mrs. Guinea lost her husband at the age of 65. She is eligible to receive $100,000 in cash for the life insurance policy of her husband. She also has the alternate choice of receiving $15,000 in annual installments for as long as she lives, with the first payment available now. Mrs. Guinea can invest money with a return of 12%, and she has chosen the second option. Find the minimum number of installments that she should receive to come out ahead. Actuarial tables indicate that the expected life of a 65-year old female is 16 years, with a standard deviation of 6 years. What is the probability that Mrs. Guinea has made the right decision?

Suppose Mrs. Guinea lives for the next n years. The first payment is available immediately. To break even,

$$100{,}000 = 15{,}000 + \sum_{i=1}^{n}\frac{15000}{1.12^i} \quad \textbf{(A)}$$

Simplifying terms,

$$85 = \frac{15\,(1-1.12^{-n})}{0.12}$$

Or, $$1.12^{-n} = 0.32$$

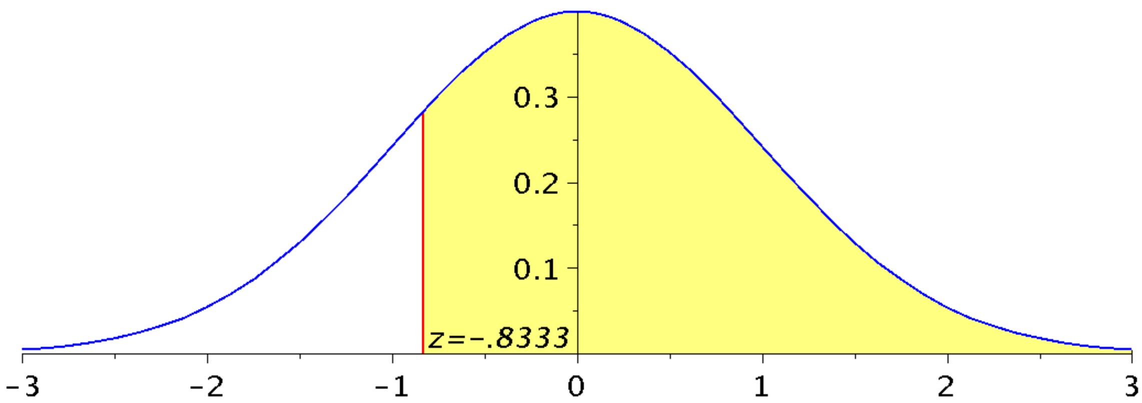
Or, $$n = -\frac{\ln(0.32)}{\ln(1.12)} = 10.05$$

You may check the answer at WolframAlpha, by writing equation **(A)** as

WRA **100000=15000+sum(15000/1.12^i,i=1..n)**

Thus, Mrs. Guinea must live another 11 years and collect 12 installments in all to come out ahead. Since she expects to live another 16 years, there is more than 50% chance that she will live another 11 years. We may find the probability that she may indeed live an additional 11 years as follows. Find

$$z = \frac{x - \mu}{\sigma} = \frac{11 - 16}{6} = -.8333$$



Draw a normal probability distribution curve, with $z = 0$ in the center and $z = -.8333$ to the left of center. We need to find the area to the right of $z = -.8333$ to find the probability that she will live more than 11 years. This area is well over 50%. Using the table, we get the probability as

$$\text{P(life} > 11) = 0.5 + 0.2967 + .33(.2995 - .2967) = 0.7976 \approx 80\%$$ ♥

EXCEL =1-NORMDIST(11,16,6,TRUE)

To do the problem on an Excel spreadsheet, enter the information as follows.

	A	B
1	Expected life, μ (years) =	16
2	Standard deviation, σ (years) =	6
3	Required life, *x* (years) =	11
4	Probability of attaining that =	=1-NORMDIST(B3,B1,B2,true)

Video 05B, 5.4. Quincy Corporation is planning to buy a machine for \$80,000. The company will depreciate it over a 5-year period with no resale value. However, the machine has an uncertain life as given in the following probability distribution table:

Probability	Life (years)
40%	4
30%	5
30%	6

While the machine is running, it will produce an EBIT of \$20,000 a year. The tax rate of Quincy is 30%, and the proper discount rate is 12%. Should Quincy buy the machine?

The annual depreciation of the machine is \$16,000 and its tax benefit is .3(16,000) = \$4800. The after-tax cash flow, C = 20,000(1 – .3) + .3(16,000) = \$18,800.

If the machine breaks down at the end of the fourth year, the company can take the tax-benefit of depreciation of the fifth year at that time. The cash flows are \$18,800 for the years 1–4, plus another \$4800 for year 4.

If the machine runs for five years, the cash flow for each year is \$18,800.
If the machine runs for six years, the cash flows for the first five years are \$18,800 each. For the sixth year, the tax benefit of depreciation is not available, and the cash flow is only 20,000(1 – .3) = \$14,000.

Including the probability of each outcome, we get

$$\text{NPV} = -80{,}000 + .4\left[\sum_{i=1}^{4}\frac{18{,}800}{1.12^i} + \frac{4800}{1.12^4}\right] + .3\sum_{i=1}^{5}\frac{18{,}800}{1.12^i} + .3\left[\sum_{i=1}^{5}\frac{18{,}800}{1.12^i} + \frac{14{,}000}{1.12^6}\right] \quad \textbf{(A)}$$

$$= -80{,}000 + 24{,}061.06 + 20{,}330.94 + 22{,}458.79 = -\$13{,}149, \text{ reject } \heartsuit$$

You may check the answer at WolframAlpha, by writing equation **(A)** as

```
.4*(sum(18800/1.12^i,i=1..4)+4800/1.12^4)+.3*sum(18800/1.12^i,i
=1..5)+.3*(sum(18800/1.12^i,i=1..5)+14000/1.12^6)-80000
```

5.5. Cavendish Company wants to acquire a new computer that will cost \$150,000 and it will save the company \$33,000 annually. The following probability distribution table represents the best estimate of its expected useful life and the corresponding salvage value:

Probability	Life	Resale Value
20%	4 years	\$30,000
30%	5 years	\$20,000
50%	6 years	\$10,000

The company will depreciate the computer fully in 4 years. The tax rate of Cavendish is 30%, and the proper discount rate is 8%. Should Cavendish buy the computer?

The tax benefit of the computer for each year is tD = .3(150,000/4) = $11,250. The present value of the tax benefits for all four years is thus

$$\sum_{i=1}^{4} \frac{11{,}250}{1.08^i} = \$37{,}261.43$$

Next, look at the probability of different lives, savings generated by the computer, and the salvage value in each case. Since the company will depreciate the computer completely when it sells it, the proceeds of the sale are fully taxable. After taxes, the resale value after four years is 30,000(1 − .3) = $21,000, after five years it is 20,000(1 − .3) = $14,000, and after six years it is 10,000(1 − .3) = $7000.

The annual earnings after taxes, excluding the tax benefit of depreciation, are 33,000(1 − .3) = $23,100. Combining all the above numbers, we find the NPV as follows:

$$\text{NPV} = -150{,}000 + 37{,}261.43 + 0.2\left(\sum_{i=1}^{4} \frac{23{,}100}{1.08^i} + \frac{21{,}000}{1.08^4}\right) + 0.3\left(\sum_{i=1}^{5} \frac{23{,}100}{1.08^i} + \frac{14{,}000}{1.08^5}\right)$$
$$+\, 0.5\left(\sum_{i=1}^{6} \frac{23{,}100}{1.08^i} + \frac{7000}{1.08^6}\right) \qquad \textbf{(A)}$$

$$= -\$8{,}221.64, \text{ reject } \heartsuit$$

You may check the answer at WolframAlpha, by writing equation **(A)** as

```
sum(.3*150000/4/1.08^i,i=1..4)+.2*(sum(23100/1.08^i,i=1..4)
+21000/1.08^4)+.3*(sum(23100/1.08^i,i=1..5)+14000/1.08^5)+.
5*(sum(23100/1.08^i,i=1..6)+7000/1.08^6)-150000
```

5.6. Jupiter Company needs a new computer costing $100,000. Jupiter will depreciate the computer over 5 years with no resale value. There is, however, a 30% chance that it may break down completely after 4 years. While the computer is running, it will add $40,000 annually to the pretax income of Jupiter, which has a tax rate of 40%. For a discount rate of 8%, should Jupiter buy this computer?

Here C = 40,000(1 – .4) + .4(20,000) = $32,000

If the machine breaks down after 4 years, we have to take the fifth-year depreciation at that time. The probability of breakdown after four years is 30% and there is a 70% probability that it would run smoothly for 5 years. Thus

$$\text{NPV} = -100{,}000 + .3\left[\sum_{i=1}^{4} \frac{32{,}000}{1.08^i} + \frac{.4(20{,}000)}{1.08^4}\right] + .7\left[\sum_{i=1}^{5} \frac{32{,}000}{1.08^i}\right] \qquad \textbf{(A)}$$

= \$22,997, buy it. ♥

You may check the answer at WolframAlpha, by writing equation **(A)** as

```
.3*(sum(32000/1.08^i,i=1..4)+.4*20000/1.08^4)+.7*sum(32000/
1.08^i,i=1..5)-100000
```

Video 05.07, 5.7. Fisher Corporation does not expect to pay taxes in the near future. It is planning to acquire a new machine, which will have a useful life of two years. However, there is a 10% probability that the machine will break down after only one year. There are two states of economy, good and bad, with the probability of occurrence 60% and 40% in any given year. If the economy is good, the after-tax cash flow from the machine is \$20,000 annually, and if the economy is bad, the cash flow is only \$15,000. The proper discount rate is 12%. What is the maximum amount that Fisher should pay for this machine?

The company is not paying taxes, thus $t = 0$. Using (4.3),

$$C = E(1 - t) + tD \tag{4.3}$$

we get $C = E$, meaning that the cash flows before and after taxes are identical. We find the expected cash flow under two different states of the economy by multiplying the probability by the corresponding cash flow and adding the results. This gives us

$$\text{E}(C) = .6(20{,}000) + .4(15{,}000) = \$18{,}000$$

Next, consider two possible outcomes of the life of the project. To get the total NPV, multiply each probability of life with the dollar outcome of that life, whether it is one year or two years. To break even, we get

$$\text{NPV} = -I_0 + .1\left(\frac{18{,}000}{1.12}\right) + .9\left(\frac{18{,}000}{1.12} + \frac{18{,}000}{1.12^2}\right) = 0 \qquad \textbf{(A)}$$

Or, $$I_0 = .1\left(\frac{18{,}000}{1.12}\right) + .9\left(\frac{18{,}000}{1.12} + \frac{18{,}000}{1.12^2}\right) = \$28{,}986$$ ♥

You may check the answer at WolframAlpha, by writing equation **(A)** as

WRA
```
-x+.1*18000/1.12+.9*(18000/1.12+18000/1.12^2)=0
```

5.8. Quincy Corporation wants to buy a machine for \$50,000, with a maximum life of 4 years. However, there is a 20% probability that the machine will break down after only 3 years. There is an investment tax credit of 6%. The company will depreciate the machine on *ACRS* basis, with a life of 3 years. Assume that the depreciation in the first year is 25%, second year 38%, and 37% in the third year. The tax rate of the company is 34%

and its discount rate is 12%. What is the minimum pretax income of this machine to make it profitable for Quincy?

Because of the 6% investment tax credit, the net cost of the equipment is 94% of its price, namely, 0.94(50,000) = $47,000. The company can depreciate only this amount. The tax benefit of depreciation is *tD* per year.

The PV of tax benefits of depreciation = $.34(47{,}000)\left(\frac{.25}{1.12} + \frac{.38}{1.12^2} + \frac{.37}{1.12^3}\right)$ = $12,616.32

Suppose the minimum pre-tax earning is E, which is just enough to break even. Its after-tax value is $E(1 - .34) = .66E$. There are two possible outcomes: the life of the machine is either three years (probability 20%) or four years (probability 80%). Including the probability of breakdown, we get

$$\text{PV of earnings} = .2\sum_{i=1}^{3}\frac{.66E}{1.12^i} + .8\sum_{i=1}^{4}\frac{.66E}{1.12^i} = \left[\frac{.2(.66)(1-1.12^{-3})}{.12} + \frac{.8(.66)(1-1.12^{-4})}{.12}\right]E$$

$$= 1.920762\,E$$

To break even, the NPV is equal to zero. Thus

$$-47{,}000 + 12{,}616.32 + 1.920762\,E = 0$$

This gives $E = \$17{,}901$ ♥

You may check the answer at WolframAlpha, by using the following instruction.

```
47000=.34*47000*(.25/1.12+.38/1.12^2+.37/1.12^3)+.2*sum(.66
*x/1.12^i,i=1..3)+.8*sum(.66*x/1.12^i,i=1..4)
```

Video 05.09, **5.9.** Galen Mining requires a digging machine that costs $50,000. It will depreciate the machine uniformly over its life of 5 years. The tax rate of Galen is 35% and the proper discount rate is 11%. Because of the uncertain price of the ore, the expected pre-tax revenue from the machine is $15,000 annually, with a standard deviation of $3,000. What is the probability that the machine will prove to be profitable?

First, find the income generated by the machine that is just enough to break even, or make NPV = 0. If the after-tax cash flow is C, then to break even

$$\text{NPV} = 0 = -50{,}000 + \sum_{i=1}^{5}\frac{C}{1.11^i}$$

Or, $$50{,}000 = \frac{C(1-1.11^{-5})}{.11}$$

This gives $$C = \frac{50{,}000(.11)}{1 - 1.11^{-5}} = \$13{,}528.52$$

To verify the result, use the following at WolframAlpha,

WRA **0=-50000+sum(x/1.11^i,i=1..5)**

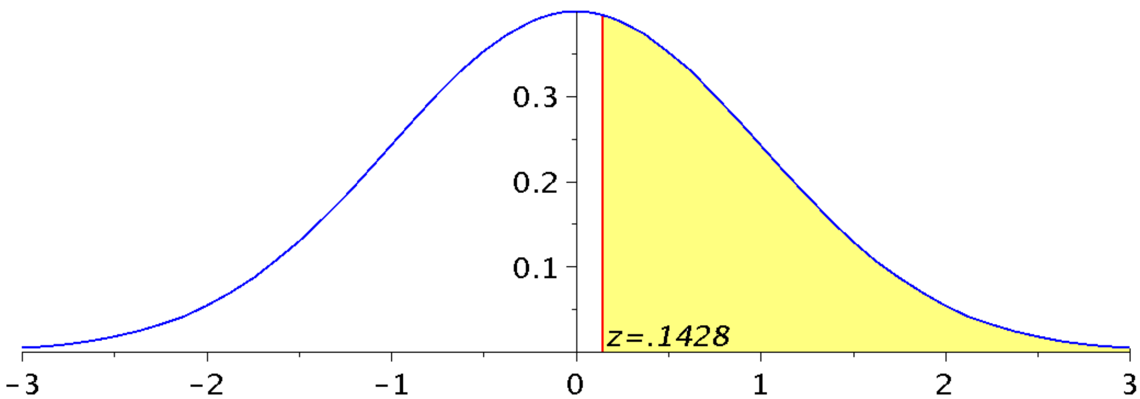
The annual depreciation is \$10,000. If E is the corresponding pre-tax earnings, then

$$13528.52 = E\,(1 - .35) + .35(10{,}000)$$

This gives $$E = \$15{,}428.49.$$

The machine must generate \$15,428.49 to break even. Since it is expected to make only \$15,000 annually, chances are less than 50% that it will be profitable. Further,

$$z = (15{,}428.49 - 15{,}000)/3000 = .1428$$



Draw a normal probability distribution curve, with z = 0 in the center and z = .1428 somewhat right of the center. The area further to the right of z = .1428, under the tail of the curve, gives the answer. Checking the tables, we get

$$\text{P(being profitable)} = 0.5 - [.0557 + .28(.0596 - .0557)] = .4432 = 44.32\% \; \heartsuit$$

You may verify the answer by using the following instruction at Exce.

EXCEL =1-NORMDIST(15428.56,15000,3000,TRUE)

5.10. Adams Corporation is planning to buy a machine that will cost \$40,000 and depreciate it on a straight-line basis over a 5-year period with no residual value. The tax rate of Adams is 30%, and the proper discount rate is 15%. The earnings before taxes from the machine are uncertain, but their expected value is \$15,000 a year, with a standard deviation of \$3,000. Calculate the probability that the machine will be a profitable investment. Adams requires the probability of being profitable to be more than 60% to buy the machine. Based on your calculation, should Adams buy the machine?

First, find the break-even point, where the earnings before taxes E are just enough to make the NPV of the investment equal to zero.

After-tax cash flow, $C = E(1 - .3) + .3(8000) = .7E + 2400$

At break even point, $\text{NPV} = 0 = -40{,}000 + \sum_{i=1}^{5} \frac{.7E + 2400}{1.15^i}$

WRA **0=-40000+sum((.7*x+2400)/1.15^i,i=1..5)**

This gives $E = \$13{,}618.$

Since the breakeven point is $13,618 and the machine is expected to make $15,000, chances are more than 50% that the machine will be profitable.

Further $z = (15000 - 13{,}618)/3000 = 0.4607$

From the tables, P(profitable) = 0.5 + .1772 + 0.07(0.1808 – 0.1772) = 67.75%

You can get the same answer at Excel by using the following instruction.

EXCEL =1-NORMDIST(13618,15000,3000,TRUE)

Since Adams requires the probability of profitability to be at least 60%, it should buy the machine. ♥

Video 05.11, 5.11. Benton Corporation is planning to buy a machine that may add $4000 to the pre-tax earnings of the company if the economy is good (probability 60%), or only $3500 if the economy is bad (probability 40%). Benton will depreciate the machine on the straight-line basis for four years, even though it has a 20% chance that it may last for five years. The tax rate of Benton is 30%, and the proper discount rate is 11%. Find the maximum price that Benton should pay for this machine to make it a profitable investment.

Suppose the break-even price of the machine is P and thus the depreciation per year is $P/4$. The expected pretax earnings are 0.6(4000) + 0.4(3500) = $3800. The after-tax earnings are

$C = 3800(1 - .3) + .3(P/4) = 2660 + .075P$, for the years 1-4
$C = 3800(1 - .3) = \$2660$ for the fifth year.

To break even, the present value of the machine should be equal to the present value of all cash flows. Including the 20% probability for the fifth-year cash flow, we get

$$P = \sum_{i=1}^{4} \frac{2660 + .075P}{1.11^i} + \frac{.2(2660)}{1.11^5} \quad \textbf{(A)}$$

Or, $$P\left(1 - \sum_{i=1}^{4} \frac{.075}{1.11^i}\right) = \sum_{i=1}^{4} \frac{2660}{1.11^i} + \frac{.2(2660)}{1.11^5}$$

Or, $$.7673165733\, P = 8252.505534 + 315.7161065$$

Solving for P, we get $P = \dfrac{8252.505534 + 315.716106}{.7673165733} = \$11{,}166.$

If Benton buys the machine for less than $11,166, it should be a profitable investment. ♥

To verify the answer at WolframAlpha, write equation **(A)** as,

WRA **`P=sum((2660+.075*P)/1.11^i,i=1..4)+.2*2660/1.11^5`**

Video 05.12, 5.12. Hawley Corporation needs a new computer that will produce annual savings estimated at $5000 with a standard deviation of $2000. The company will buy the computer for $20,000, depreciate if fully on a straight line over four years, and then sell it for $3,000. The tax rate of Hawley is 25%, and it uses a discount rate of 11%. Find the probability that the computer will have a positive NPV. Should Hawley buy the computer?

In this problem, the additional factor to consider is the resale value of the machine. To calculate the after-tax value of the resale price, we use

$$W = S(1 - t) + tB \tag{4.7}$$

Since the machine is fully depreciated, its book value B is zero, Hawley has to pay taxes on the sales price of the machine. After taxes, it becomes $(1 - .25)(3000) = \$2250$.

To break even, suppose the earnings before taxes are E. With $5000 annual depreciation and 25% tax rate, the after-tax cash flow is, by (4.3),

$$C = E(1 - .25) + .25(5000) = .75E + 1250$$

This gives

$$\text{NPV} = 0 = -20{,}000 + \sum_{i=1}^{4} \frac{.75E + 1250}{1.11^i} + \frac{2250}{1.11^4} \quad \textbf{(A)}$$

$$20{,}000 - \sum_{i=1}^{4} \frac{1250}{1.11^i} - \frac{2250}{1.11^4} = E \sum_{i=1}^{4} \frac{0.75}{1.11^i}$$

Or, $$E = \$6291.72$$

To verify the result at WolframAlpha, use the following instruction,

WRA **0=-20000+sum((.75*x+1250)/1.11^i,i=1..4)+2250/1.11^4**

The computer is expected to generate only $5000 annually. Thus is it unlikely that it will be profitable. To visualize that, draw a normal probability distribution curve, with μ = $5000 at the center and σ = $2000. The required x = $6291.72 is somewhat to the right of center. The area further to the right of x gives the answer. Calculate the z-value as

$$z = \frac{6291.72 - 5000}{2000} = 0.6459$$

From the tables,

$$P(NPV > 0) = 0.5 - [0.2389 + 0.59(0.2422 - 0.2389)] = 25.92\%$$

You may check the answer at Excel by using the following expression,

EXCEL =1-NORMDIST(6291.72,5000,2000,TRUE)

The probability that the computer is going to be profitable is only about 26%. Hawley should not buy it. ♥

5.13. Columbus Corporation plans to acquire a computer that will last for 5 years, and costs $100,000. The company will use straight-line method to fully depreciate the computer in 4 years, and then sell it for $10,000 after 5 years. Columbus will use 9% discount rate for this investment, and its income tax rate is 35%. The pretax saving from this computer is uncertain, with expected value $24,000 per year, and standard deviation $6,000. What is the probability that the computer will have a positive NPV? Should Columbus buy this machine?

First we find the pretax earnings E that will make NPV = 0. The annual depreciation is $25,000. The after-tax cash-flow is given by (4.3)

$$C = E(1 - .35) + .35(25{,}000) = .65E + 8750$$

Consider this as two cash flows: .65E, which will go on for five years and $8750 only for the first four years. Finally, the after-tax value of the sales price is (1 – .35)10,000 = $6500. Setting the NPV equal to zero, we get

$$0 = -100{,}000 + \sum_{i=1}^{4} \frac{8750}{1.09^i} + \sum_{i=1}^{5} \frac{.65E}{1.09^i} + \frac{6500}{1.09^5} \quad \textbf{(A)}$$

Or, $$0 = -100{,}000 + 28{,}347.54 + 2.52827E + 4224.55$$

This gives $$E = \$26{,}669.54$$

Verify the answer by solving **(A)** at WolframAlpha as follows,

WRA **100000=sum(8750/1.09^i,i=1..4)+sum(.65*x/1.09^i,i=1..5)+6500/1.09^5**

Since the machine is expected to generate \$24,000 annually in earnings, it is unlikely that it will become profitable. To visualize that, draw a normal probability distribution curve, with μ = \$24,000 at the center and σ = \$6000. The required x = \$26,669.54 is somewhat to the right of center. The area further to the right of x gives the answer. Calculate the z-value as

$$z = (26{,}669.54 - 24{,}000)/6{,}000 = .4449$$

From the tables,

$$\text{P(profitable)} = .5 - [.1700 + .49(.1736 - .1700)] = 32.82\%$$

You can also get the same result by using Excel as,

EXCEL =1-NORMDIST(26669.54,24000,6000,TRUE)

Considering the low probability of profitability, Columbus should not buy it. ♥

5.14. Lucas Corporation plans to buy a machine for \$100,000, and depreciate it uniformly over its useful life of 5 years. The machine will produce annual pretax revenue of \$27,000. The tax rate of Lucas is 35% and it will use 11% as the discount rate for this investment. The resale value of the machine after five years has a mean of \$20,000 with a standard deviation of \$5000. Calculate the probability that this machine will be profitable. Should Lucas buy it?

Suppose the resale value of the machine is S to break even. Since the machine is fully depreciated at the time of sale, the sale amount is taxable. Its after-tax value is $(1 - .35)S = .65S$. The annual depreciation is \$20,000, and thus the annual after-tax cash flow from this machine is

$$C = (1 - .35)27{,}000 + .35(20{,}000) = \$24{,}550$$

The gives

$$\text{NPV} = 0 = -100{,}000 + \sum_{i=1}^{5} \frac{24{,}550}{1.11^i} + \frac{.65S}{1.11^5} \quad \textbf{(A)}$$

Solve **(A)** at WolframAlpha as follows,

WRA **100000=sum(24550/1.11^i,i=1..5)+.65*S/1.11^5**

to get S, S = \$24,020.45

Since the resale value is only \$20,000, the probability is less than 50% that it will be profitable. Draw a normal probability distribution curve, with μ = \$20,000 at the center and σ = \$5000. The required x = \$24,020.45 is somewhat to the right of center. The area further to the right of x gives the answer. Calculate the z-value as

$$z = (24020 - 20000)/5000 = .8041$$

Using the tables,

$$\text{P(profitable)} = .50 - [.2881 + .41(.2910 - .2881)] = .2107$$

EXCEL =1-NORMDIST(24020.45,20000,5000,TRUE)

There is only a 21% probability that the machine will be profitable. The company should not buy it. ♥

5.15. Delta Corporation is planning to buy a new machine at a cost of \$30,000, which will increase the pretax earning of the company by \$10,000 annually. The maximum life of the machine is 5 years, but there is also a 10% probability that it will break down after 3 years and a 20% probability that it will last only 4 years. Delta will depreciate the machine on a straight-line basis for 5 years. In the case of a breakdown, Delta will discard the machine, and lease a replacement machine for \$8,000 annually, paying it in advance every year. The after-tax cost of capital is 12%, and the company is in 30% tax bracket. Should Delta proceed with the purchase? It cannot lease the machine for full five years.

First, look at the earnings of the machine. The pretax earning from the machine, whether it is the first one or the leased one, is \$10,000 a year and its after-tax value is 10,000(1 − .3) = \$7,000. Take the PV for the full five years. This amounts to

$$\text{PV of } E(1-t) = \sum_{i=1}^{5} \frac{7000}{1.12^i} = \$25{,}233.43 \qquad \textbf{(A)}$$

You may solve **(A)** at WolframAlpha as follows,

WRA **`sum(7000/1.12^i,i=1..5)`**

Next, consider the possibility that the machine breaks down after 3 years. The company will write it off and lease a second one. At that time, the company does the following:

1. Takes the tax benefits of depreciation for the fourth and the fifth year. The depreciation per year is \$6,000 and its tax benefit, tD, is .3(6000) = \$1800 per year. The total tax benefit for the fourth and the fifth years is \$3600.

2. Pays the lease payments for the two years in advance each year. The lease payments are tax deductible, and so their after-tax value is 8000(1 – .3) = \$5600 annually.

3. Considers the 10% probability that the machine breaks down after three years and finds the PV of the cash flows.

$$\text{PV of 3-year life} = .1\left(\sum_{i=1}^{3}\frac{1800}{1.12^i}+\frac{3600}{1.12^3}-\frac{5600}{1.12^3}-\frac{5600}{1.12^4}\right) = -\$65.92 \qquad \textbf{(B)}$$

You may solve **(B)** at WolframAlpha as follows,

WRA `.1*(sum(1800/1.12^i,i=1..3)+3600/1.12^3-5600/1.12^3-5600/1.12^4)`

In the above expression, the summation is the PV of tax benefit of depreciation for the first three years; $3600 is the tax benefit for years 4 and 5; $5600 is the after-tax value of lease payment; and 0.1 is the probability factor.

Now consider the possibility that the machine runs for four years. At the end of the fourth year, the company does the following:

1. Finds the PV of four years of tax benefits of depreciation, $1800 per year.

2. Takes the fifth year depreciation at the end of the fourth year. Its tax benefit is $1800.

3. Pays the lease payment for the fifth year, $5600 after taxes.

4. Incorporates the 20% probability of this event and calculates the PV of the cash flows.

$$\text{PV of 4-year life} = .2\left(\sum_{i=1}^{4}\frac{1800}{1.12^i}+\frac{1800}{1.12^4}-\frac{5600}{1.12^4}\right) = \$610.45 \qquad \textbf{(C)}$$

You may solve **(C)** at WolframAlpha as follows,

WRA `.2*(sum(1800/1.12^i,i=1..4)+1800/1.12^4-5600/1.12^4)`

The PV of a 5-year life for the first machine does not involve the second machine. The probability of this event is 70%. Thus

$$\text{PV of 5-year life} = .7\left(\sum_{i=1}^{5}\frac{1800}{1.12^i}\right) = \$4542.02 \qquad \textbf{(D)}$$

You may solve **(D)** at WolframAlpha as follows,

WRA `.7*sum(1800/1.12^i,i=1..5)`

Combine (A), (B), (C), and (D) to get the *NPV* of all possibilities

NPV = – 30,000 + 25,233.43 – 65.92 + 610.45 + 4,542.02 = $319.98.

The machine is barely acceptable. ♥

5.2 Inflation

In financial modeling, one can also include the effects of inflation, which will influence the future earnings or expenses of a corporation. Although inflation rates are difficult to predict accurately, one can use the past as a proxy for the future and arrive at a reasonable estimate of the rate of inflation. We should adjust the future cash flows accordingly.

Examples

5.16. A corporation has the following pension benefit for its retired employees. It pays annual payments equal to 35% of an employee's last salary at the time of retirement, adjusting them upward according to the rate of inflation. Find the cost of these benefits to the company for an employee who has just retired with an annual salary of $32,000. The company expects to make 16 annual payments and expects the inflation rate to be 3.5% during this period. The cost of capital to the company is 12%. The company will make the first payment now.

First payment = 0.35(32,000) = $11,200

$$\text{NPV} = 11{,}200 + \sum_{i=1}^{15} \frac{11{,}200(1.035)^i}{1.12^i} \qquad \textbf{(A)}$$

$$= 11200 + \frac{11{,}200\ (1.035)}{1.12} + \frac{11{,}200\ (1.035)^2}{1.12^2} + \ldots + \frac{11{,}200\ (1.035)^{15}}{1.12^{15}}$$

In this summation, $a = \dfrac{11{,}200\ (1.035)}{1.12}$, $x = \dfrac{1.035}{1.12}$, and $n = 15$. Using (1.4), we get

$$\text{NPV} = 11{,}200 + \frac{11{,}200\ (1.035)}{1.12}\left[\frac{1 - (1.035/1.12)^{15}}{1 - 1.035/1.12}\right] = \$105{,}834$$

The company may also offer to make a single payment of $105,834 as the settlement for the pension benefits. ♥

You may solve **(A)** at WolframAlpha as follows,

WRA **`11200+sum(11200*1.035^i/1.12^i,i=1..15)`**

5.17. Benin Corporation has the following pension plan. It will give 35% of a person's last annual salary as pension in annual installments, with the first payment at the time of retirement. The pension has a cost of living adjustment, which is expected to be +3% annually in the future. Benin uses a discount rate of 9%. Find the present value of the pension cost for an employee who has just retired with an annual salary of $42,000. She expects to receive 17 payments.

$$PV = 0.35(42{,}000) + \frac{0.35(42{,}000)(1.03)}{1.09} + \frac{0.35(42{,}000)(1.03)^2}{1.09^2} + 17 \text{ terms } \textbf{(A)}$$

$$= \frac{0.35(42{,}000)[1 - (1.03/1.09)^{17}]}{1 - 1.03/1.09} = \$165{,}056 \ ♥$$

You may solve **(A)** at WolframAlpha as follows,

WRA `sum(.35*42000*(1.03/1.09)^i,i=0..16)`

5.18. California Gold Mining Company plans to buy a dredging machine, which costs $40,000. The machine will produce an EBIT of $4,000 in the first year, but this will increase at the rate of 10% annually due to the rising price of gold. The machine will last for 10 years and the company will depreciate it on a straight-line basis. The tax rate of the company is 30% and the proper discount rate is 12%. Should California buy the machine?

Suppose the initial investment in the machine is I_0. The depreciation per year is thus I_0/n, and the tax benefit of depreciation per year is $t(I_0/n)$. Suppose the EBIT from the machine in the first year is E. Due to the inflation rate of f per year, it will become $E(1 + f)$ in the second year, $E(1 + f)^2$ in the third year, and so on. To generalize, the earnings in the ith year are $E(1 + f)^{i-1}$. Thus

$$\text{NPV} = -I_0 + \sum_{i=1}^{n} \frac{E(1+f)^{i-1}(1-t) + tI_0/n}{(1+r)^i}$$

Substituting the numerical values, I_0 = $40,000, E = $4,000, $f = 0.1$, $n = 10$, $t = 0.3$, $r = 0.12$, we get

$$\text{NPV} = -40{,}000 + \sum_{i=1}^{10} \frac{4000(1.1)^{i-1}(1-.3) + .3(40{,}000)/10}{1.12^i}$$

Use the Maple expression (red) or WolframAlpha (blue) to get the result.

```
-40000+sum((4000*.7*1.1^(i-1)+.3*40000/10)/1.12^i,i=1..10);
```

```
-40000+sum((4000*.7*1.1^(i-1)+.3*40000/10)/1.12^i,i=1..10)
```

The result is NPV = − $10,135.92, which is not acceptable. ♥

5.19. Carpenter Corporation is considering a project with an initial cost of $100,000. The pre-tax cash inflows in current dollars, without adjustment for inflation, are as follows.

Year:	1	2	3	4	5
Amount:	$40,000	$30,000	$30,000	$30,000	$40,000

The estimate for inflation during this period is that it will be either 4%, with probability 30%; or 5%, with probability 70%. Accordingly, the company should adjust the cash flow projections upwards. The company is in 35% tax bracket and it uses a discount rate of 12%. Should it accept the project?

Assuming that the inflation is 4% annually, the pretax cash flows will become 40,000(1.04), $30{,}000(1.04)^2$, $30{,}000(1.04)^3$, etc. We multiply the cash flows by (1 – .35) to get their after-tax amount. Multiplying with the respective probabilities, and discounting, we get the NPV as

$$\text{NPV} = -\,100{,}000 + (1 - .35) \times$$

$$\left\{0.3\left[\frac{40{,}000(1.04)}{1.12} + \frac{30{,}000(1.04)^2}{1.12^2} + \frac{30{,}000(1.04)^3}{1.12^3} + \frac{30{,}000(1.04)^4}{1.12^4} + \frac{40{,}000(1.04)^5}{1.12^5}\right]\right.$$

$$\left. + \, 0.7\left[\frac{40{,}000(1.05)}{1.12} + \frac{30{,}000(1.05)^2}{1.12^2} + \frac{30{,}000(1.05)^3}{1.12^3} + \frac{30{,}000(1.05)^4}{1.12^4} + \frac{40{,}000(1.05)^5}{1.12^5}\right]\right\}$$

$= -\$9264$, reject the project. ♥

You may verify the result at WolframAlpha as follows,

WRA `-100000+(1-.35)*(.3*sum(.35*42000*(1.03/1.09)^i,i=0..16)`

5.3 The Option to Postpone a Project

In many situations, the investment decision of a corporation also has options embedded in it. For example, a corporation can start a project now, or wait for a year and then start the project. Therefore, the company has the option to delay the project. Later on, when the project is becoming profitable, the company has the option to expand the project. If the sales are not what they were originally anticipated, the firm may have the option to contract the project. Finally, if the project is creating a big loss, then the firm should have the option to abandon the project outright.

Each option listed above has a certain value, because it provides the company with flexibility in its planning. For example, the company can build a factory now, or build it next year. By building the factory now, the company exercises its option and thereby loses the value of that option. It is also possible to find the value of this option.

Suppose a corporation can start a project right away, or wait for a while and then decide whether to get on with the project. Consider the following simplified example.

Example

5.20. An oil company can invest $16,000 to drill a well and start producing oil. The revenue from the well is uncertain, depending on the price of oil. At present, the price of oil is $20 a barrel, but after a year, it could go up to $30 a barrel, or drop to $10 a barrel, with equal probability. Let us assume that the new price of oil will stay constant forever. The well will produce 100 barrels of oil a year forever with no cost. The oil revenues are available at the end of each year. The cost of capital is 10%. Should the company drill the well now, or wait for a year and then drill?

Suppose the company drills the well right now and starts producing the oil. The annual revenue from the well is $3000, or $1000, depending on the price of oil. Both of these outcomes are equally probable. The cash flows start a year from now and continue forever. The following expression gives the NPV,

$$\text{NPV} = -\,16{,}000 + .5\sum_{i=1}^{\infty}\frac{3000}{1.1^i} + .5\sum_{i=1}^{\infty}\frac{1000}{1.1^i} = -16{,}000 + .5[3000/.1 + 1000/.1] = \$4{,}000$$

Based on this incomplete analysis, the company should go ahead and drill the well. The answer is, however, wrong because we have ignored the value of the *option* to delay the decision for a year. The probability that the oil price will rise or drop is 50% each. Suppose the risk-free rate is 6%, then

$$\text{NPV(oil = \$30)} = .5\left(\frac{-16{,}000}{1.06} + \sum_{i=2}^{\infty}\frac{3000}{1.1^i}\right) = \$6089$$

$$\text{NPV(oil = \$10)} = .5\left(\frac{-16{,}000}{1.06} + \sum_{i=2}^{\infty}\frac{1000}{1.1^i}\right) = -\$3002$$

If the company waits for a year and then drills the well, then its choice is clear – it will drill *only* if the price of oil is $30 a barrel. For a price of $10, the NPV is negative and the company will not get into oil production. The company must wait for a year and then decide to drill.

Suppose the company did not have the option to wait. It is now or never. In that case, the company should go ahead and drill now with an NPV of $4000. Since the second NPV is $2089 more than the first NPV, this is the value of the option to wait. ♥

5.4 The Option to Abandon a Project

There may be several reasons for a company to stop a project: the market conditions have changed, the plans have changed, or the project simply is not profitable any more. The flexibility afforded by this option to continue or discontinue a project also adds value to the company. It is possible to evaluate the optimal time to replace a car, or a machine, or a whole factory. You may have to compare the NPV of abandoning versus the NPV of continuing the project. The following example will illustrate the point.

Examples

5.21. A company has decided to buy a machine for $50,000. It will depreciate the machine on a straight-line basis over its useful life of five years. The tax rate of the company is 40% and the proper discount rate 10%. The following tables gives the resale value, *S* of the machine and its pretax revenue, *E*.

Time, years	Pretax revenue, E	Resale value, S
0	$0	$50,000
1	$22,000	$38,000
2	$18,000	$27,000
3	$14,000	$17,000
4	$12,000	$8,000
5	$10,000	$0

The company has the option to keep or sell the machine at any time. What is the optimal time to replace the machine?

Recall (4.3) and (4.7) to calculate the after-tax cash flows and the after-tax value of the resale price of the machine. We augment the information in the above table as follows:

Time	E	S	$E(1-t)+tD$	B	$S(1-t)+tB$
0	$0	$50,000	$0	$50,000	$50,000
1	$22,000	$38,000	$17,200	$40,000	$38,800
2	$18,000	$27,000	$14,800	$30,000	$28,200
3	$14,000	$17,000	$12,400	$20,000	$18,200
4	$12,000	$8,000	$11,200	$10,000	$8800
5	$10,000	$0	$10,000	$0	$0

To find the optimal replacement time, we find the NPV of the machine if it is used only for one year, or two years, and so on. The NPV for all possibilities will be

$$\text{NPV(1 year)} = -50{,}000 + \frac{17{,}200}{1.1} + \frac{38{,}800}{1.1} = \$909$$

$$\text{NPV(2 years)} = -50{,}000 + \frac{17{,}200}{1.1} + \frac{14{,}800}{1.1^2} + \frac{28{,}200}{1.1^2} = \$1174 \text{ (highest)}$$

$$\text{NPV(3 years)} = -50{,}000 + \frac{17{,}200}{1.1} + \frac{14{,}800}{1.1^2} + \frac{12{,}400}{1.1^3} + \frac{18{,}200}{1.1^3} = \$858$$

$$\text{NPV(4 years)} = -50{,}000 + \frac{17{,}200}{1.1} + \frac{14{,}800}{1.1^2} + \frac{12{,}400}{1.1^3} + \frac{11{,}200}{1.1^4} + \frac{8{,}800}{1.1^4} = \$844$$

$$\text{NPV(5 years)} = -\,50{,}000 + \frac{17{,}200}{1.1} + \frac{14{,}800}{1.1^2} + \frac{12{,}400}{1.1^3} + \frac{11{,}200}{1.1^4} + \frac{10{,}000}{1.1^5} = \$1043$$

It is interesting to note that the NPV reaches a maximum after two years, equaling \$1174. Considering all these results, it is best to replace the machine after two years. ♥

The value of the option to stop at any time after the project has commenced is the difference between the maximum NPV and the NPV for full 5 years. It comes out to be 1174 – 1043 = \$131 ♥

5.22. A company can build a factory now at a cost of \$900, or wait for a year and build it at a cost of \$1200, or \$800. The probability of increased cost is 60%. The future cost depends on the pending legislation. The output from the factory will have a net cash flow of \$100 per year forever. The proper discount rate for the cash flows is 10%, while the risk free rate is 6%. Should the company build now or next year?

$$\text{NPV(build now)} = -\,900 + \sum_{i=1}^{\infty} \frac{100}{1.1^i} = \$100$$

$$\text{NPV(build next year)} = .6\left(\frac{-1200}{1.06} + \sum_{i=2}^{\infty} \frac{100}{1.1^i}\right) + .4\left(\frac{-800}{1.06} + \sum_{i=2}^{\infty} \frac{100}{1.1^i}\right) = -\$72.04$$

The company should build the factory now. ♥

5.23. A company can automate its payroll department by purchasing a computer. The computer costs \$25,000 now, but it may cost only \$20,000 next year. The company expects to save \$6500 a year during its useful life of 5 years. The discount rate is 12%, and the risk-free rate is 6%. The company uses straight-line depreciation and its tax rate is 30%. Should the company install the computer this year, or next year?

The depreciation in the first case is \$5000 annually, and in the second case \$4000 annually.

$$\text{NPV(install now)} = -25{,}000 + \sum_{i=1}^{5} \frac{6500(1-.3) + .3(5000)}{1.12^i} = -\$3191.10$$

$$\text{NPV(install next year)} = -\,\frac{20{,}000}{1.06} + \sum_{i=2}^{6} \frac{6500(1-.3) + .3(4000)}{1.12^i} = -\$361.26$$

The company should not install the computer at present. It should re-evaluate the situation after a year or two. Perhaps the price of the computer will drop futher, or the savings will increase. ♥

5.24. A company has the option of buying a machine now, or waiting for a year. If the company buys the machine now it will cost \$12,000 while the current discount rate is 10%. If the company buys the machine next year, the machine will cost \$13,000, but the discount rate will be 9%. In any case, the machine has a useful life of 5 years with no resale value. The machine will generate $4000 in annual pretax revenues. The tax rate of the company is 30%. The risk-free rate is 6%. What is the better strategy?

Considering the after-tax cash flows in each case, find the NPV as

$$\text{NPV(buy now)} = -\,12{,}000 + \sum_{i=1}^{5} \frac{4000(1-.3) + .3(2400)}{1.1^{i}} = \$1343.57$$

$$\text{NPV(wait for a year)} = -\,\frac{13{,}000}{1.06} + \sum_{i=2}^{6} \frac{4000(1-.3) + .3(2600)}{1.09^{i}} = \$511.034$$

The company should buy the machine now. The value of the option to wait is zero, because the company will not wait. An option cannot have a negative value. ♥

Problems

5.26. Webster Company stock does not pay any dividends and it sells for $55 a share. The continuously compounded expected return of the stock is .12, with standard deviation of 0.22. Find the probability that the stock will be selling for more than $60 after one year. $P(S > 60) = 55.96\%$ ♥

5.27. Green Acres common stock pays no dividend, but its expected return is 13% with a standard deviation of 30%. Currently the stock is selling for $65 a share. If you buy the stock now, what is the probability that its price next year will be between $60 and $70?

$P(60 < S < 70) = 18.46\%$ ♥

5.28. You are looking for an investment opportunity that will provide a yield of at least 10%. You estimate that the returns from the Ford Motor Company stock have normal distribution with mean 12.5% and standard deviation 8.5%. What is the probability of achieving your objective if you buy Ford stock? What is the probability that your loss will be limited to 5% of your investment? $P(r > .1) = 61.57\%$, $P(r > -.05) = 98.02\%$ ♥

5.29. Marshall Company is in financial distress. Its bonds will mature on December 31, 2010, and they pay 8% interest annually on December 31. There is a 30% chance that the company may become bankrupt in a given year. In case of bankruptcy, the company will not pay interest on the bonds for that year and will settle the claims by paying 30% of the principal at the end of the year. Suppose your required rate of return on these bonds is 14%, how much would you pay for a bond on January 1, 2008? $486.51 ♥

5.30. Benin Company 7.5s14 bonds are selling at 27. Your careful analysis reveals that the company will survive the first year. It may go bankrupt during the second year (probability 50%) or during the third year (probability 50%). Before bankruptcy, it will continue to pay the semiannual interest payments. In case of bankruptcy, the company will not pay any interest for that year, and you expect to get only 20% of the face amount of the bond, which will be available one year after the bankruptcy. For instance, if Benin goes bankrupt during the second year, you will receive the final payment at the end of the third year. Your required rate of return is 12%. Do you think you should buy Benin bonds? No, B = \$232.58 ♥

5.31. Macmillan Corporation's cumulative preferred stock pays \$5 annual dividend, the first one will be received a year from now. The cumulative feature means that if the company does not pay the dividends in a given year, it must pay the total dividends due the following year. The payment of dividends is contingent upon the earnings after taxes, and you feel that there is 90% chance that the dividends will indeed be paid in a given year. In any case, at the end of 3 years, the company will buy back the stock by paying \$50 per share to the stockholders, plus any dividends due. The discount rate for this investment is 14%. What price would you pay for a share of Macmillan stock? \$45.25 ♥

5.32. The York Company wants to buy a new stamping machine that costs \$100,000. The machine has an expected life of 10 years with a standard deviation of 2 years. York will depreciate the machine over a 10-year period. The cost of running the machine is \$10,000 annually, and it generates \$30,000 annual revenue. The company has income tax rate of 30%, and it uses 10% as the discount rate. What is the probability that the machine will break down within 8 years? If it runs for only 8 years, what is its NPV?

P(life < 8) = 15.87%, NPV = −\$6507.21 ♥

5.33. Burundi Airlines is planning to acquire a Boeing 757 at a cost of \$24 million. The plane has an uncertain life span: it may last for 6 years (probability 50%), 7 years (probability 30%), or 8 years (probability 20%). The airline will depreciate the plane on a straight-line basis with a life of 6 years, with no residual value. While the plane is flying, it will generate a pretax income of \$6 million annually. The tax rate of Burundi is 40% and its after-tax cost of capital is 9%. Should Burundi buy the new plane?

NPV = \$672,782, buy it ♥

5.34. Usfan Company is interested in buying a computer with uncertain life. The following table shows its expected life and resale value:

Expected life	Probability	Resale value
3 years	20%	\$20,000
4 years	30%	\$10,000
5 years	50%	\$5,000

The computer will save the company \$40,000 annually while it is running. Usfan will depreciate it fully on a straight-line basis in 3 years. The tax rate of Usfan is 30%, and the proper discount rate in this case is 12%. The cost of the computer is \$90,000. Should Usfan buy it? Yes, NPV = \$25,368 ♥

5.35. Pisces Corporation is planning to buy a machine for \$20,000 that may run for 4 years (probability 60%), 5 years (probability 30%), or for 6 years (probability 10%). Pisces will depreciate the machine on straight-line basis for 4 years, without any resale value. The tax rate of Pisces is 30% and the proper discount rate is 8%. Calculate the minimum annual earnings generated by this machine to be acceptable to Pisces.

$5887.52 ♥

5.36. Thales Corporation wants to buy a machine for $60,000. It will depreciate the machine over a six-year period with no salvage value. The machine will generate pre-tax revenue of $20,000 annually, the tax rate of Thales is 32%, and the proper discount rate is 11%. The machine has uncertain life: it may run for 5 years (probability 50%), 6 years (probability 30%), or 7 years (probability 20%). Should Thales buy the machine?

NPV = $8841.69, yes. ♥

5.37. Cobb Company is planning to invest in a machine that will run for 5 years. Cobb uses straight-line depreciation. The annual pretax revenue from the machine has normal distribution, with mean $12,000 and standard deviation $4000. The tax rate of the company is 30% and its after-tax cost of capital is 8%. How much should Cobb invest in this machine so that there is greater than 80% probability that it will turn out to be profitable. $31,731.46 ♥

5.38. Rayburn Corporation is considering the purchase of a new machine that has an expected life of 5 years with a standard deviation of 2 years. The machine will cost $40,000 and will generate a pre-tax income of $10,000 annually. The tax rate of Rayburn is 40% and its cost of capital is 7%. Rayburn will depreciate the machine on a straight-line basis over 5 years with no residual value. Calculate the probability that the machine will have a life of between 4 and 7 years. Is the machine acceptable if it runs for only 4 years? 53.28%, NPV = –$6396.48, no ♥

5.39. Acheson Corporation is planning to buy a machine that will cost $30,000. Its life is uncertain: it may become obsolete after two years (probability 25%) and definitely after three years. The economy in any given year may be good (probability 60%) or bad (probability 40%). If the economy is good, the machine will generate pre-tax annual revenue of $15,000 and if the economy is bad, the expected revenue will only be $10,000. The tax rate of Acheson is 40% and it will depreciate the machine for 3 years on a straight-line basis. The proper discount rate is 7%. Should Acheson buy the machine?

NPV = – $567.71, no ♥

5.40. Martin Company is looking into a machine with a cost of $20,000 that will run for 5 years. Martin will depreciate the machine completely over this period. The tax rate of the company is 30%, and its cost of capital is 8%. The expected pretax income from the machine is $5,000 annually, with a standard deviation of $2,000. Calculate the probability that the machine will turn out to be profitable. 41.26% ♥

5.41. Cooper Inc. plans to buy a new machine for $25,000 for a project that will last 5 years. The machine will generate $8,000 annually in pretax revenue. The tax rate of Cooper is 30% and its after-tax cost of capital 8%. Assume straight-line depreciation for five years. There is a 10% chance that the machine may break down after 3 years and a 20% probability of breakdown after 4 years. If the machine breaks down, Cooper will get a used machine as a replacement for $12,000. The used machine has a two-year useful life. At the end of the project, neither machine will have a resale value. Should Cooper buy this machine? NPV = $1418.54, buy ♥

5.42. Salam Corporation would like to buy a new electric furnace for $42,000 that will increase the EBIT of the company by $8,000 annually. The actual life of the furnace is unpredictable, but for depreciation purposes, it is 7 years. The proper discount rate is 9% and the tax rate is 30%. Find the minimum number of years that the furnace must run before it would become a profitable investment. NPV(9 years) = $632.70 > 0 ♥

5.43. Richardson Company is considering the purchase of a machine that it expects will run for 5 years, even though there is a 20% chance that it may break down completely after 4 years. While the machine is running, it will generate $20,000 annually in pre-tax income. Richardson will depreciate the machine on a straight-line basis over a five-year period with no salvage value. The income tax rate of Richardson is 30% and its after-tax cost of capital is 11%. The cost of the machine is $80,000. Should Richardson install the machine? NPV = – $12,116, no ♥

5.44. Laird Company is planning to invest in a project with expected after-tax cash flow of $12,000 annually with a standard deviation $4000. The project requires an initial investment of $75,000 and another expense of $25,000 at the end of the first year. The first income will occur at the end of the second year. Calculate the probability that the project will be profitable (that is, its NPV > 0) after 10 years. The cost of capital to Laird is 9%. 7.33% ♥

5.45. Bush Inc. is planning to acquire a machine with expected life 5 years. Bush will depreciate the machine on a straight-line basis without any salvage value. The following table shows the pre-tax income generated by the machine depending on the state of the economy.

State of the economy	Probability	Expected income
Good	30%	$23,000
Fair	50%	$18,000
Poor	20%	$12,000

The economic conditions in any given year are independent of the conditions in the previous year. Bush will not buy the machine unless it has NPV of at least $30,000. The proper discount rate in this case is 12%, and the income tax rate for Bush is 35%. Calculate the maximum price that Bush should pay for this machine. $17,225.36 ♥

5.46. Benue Company is interested in buying a machine that costs $80,000. Benue will depreciate the machine over a 5-year period with no resale value. The actual life of the machine is uncertain, but while it is operating, it will generate pretax revenue of $25,000 annually. The tax rate of Benue is 30%, and it uses 10% discount rate for such an investment. Find the NPV of this investment if the machine lasts (a) 4 years, and (b) 7 years. (a) – $6033.54, (b) $23,393.11 ♥

5.47. Bomu Corporation is planning to buy a $70,000 machine with a 5-year life. Bomu will depreciate the machine fully over that period and then sell it for $10,000. The annual pretax revenue from the machine is uncertain, with a mean of $25,000, and standard deviation $10,000. The income tax rate of Bomu is 30% and the cost of capital 12%. Find the probability that the machine will be profitable. 68.56% ♥

5.48. Syracuse Company is planning to buy a machine for $50,000 that will be depreciated fully in five years on a straight-line basis. The machine is estimated to last 7 years, and then it will be sold for $5000. The before-tax earnings from the machine are estimated to be $10,000 annually, with a standard deviation of $2000. The tax rate of Syracuse is 30%, and its after-tax cost of capital 12%. Find the probability that this machine will be profitable. 18.8% ♥

5.49. Cleveland Corporation needs a machine that will cost $120,000 and it will generate $25,000 annual pretax earnings. The company will depreciate the machine over 5 years, with no resale value. The discount rate for this investment is 8%, and the income tax rate of Cleveland is 32%. The machine may actually run for 5 years (probability 30%), or 6 years (probability 70%). Should Cleveland buy this machine? No, NPV = −$13,961 ♥

5.50. Chadwick Company has a pension plan that provides lifetime benefits to its retiring employees, including cost of living adjustments. The first payment, paid a year after the retirement, is equal to 60% of the last annual salary. However, the benefits are expected to rise by 4% annually in the future. The expected number of annual payments to a 65-year old person is 18. The cost of capital to Chadwick is 11%. Find the present value of the benefits payable to a person whose last salary is $50,000 per year. $295,890.13 ♥

5.51. Lakeland Clinic, a tax-exempt entity, is interested in buying a MRI device now, or postponing it for a year. The current price of the equipment is $3 million, but it is expected to go up to $3.2 million next year. The hospital plans to use the machine for a period of 6 years, and during this period the value of the machine is expected to drop at a compound annual rate of 10%. The equipment will produce revenue of $500,000 annually. The cost of capital for hospital is 8%. What is the best course of action? What is the value of the option to wait?
NPV(now) = $316,134, NPV(later) = $169,549, option value = 0 ♥

5.52. Western Airlines is planning to acquire a plane for $50 million. They will depreciate it completely on a straight-line basis over 5 years. The tax rate of Western is 30% and its cost of capital 10%. The plane should generate $20 million annually in pretax income, not including maintenance costs. The resale value and the annual maintenance cost (in millions) of the plane are variable as shown in the following table:

Time	Maintenance	Resale value
0 year	$0	$50 million
1 year	$5 million	$45 million
2 year	$8 million	$40 million
3 year	$10 million	$35 million

4 year	$12 million	$30 million
5 year	$15 million	$25 million

What is the optimal time to keep the airplane flying? What is the value of the option to sell the plane ahead of time, rather than to keep it for full five years?

Optimal 2 years, NPV(2) = $2.273 million, value of the option = $2.289 million ♥

Key Terms