3. VALUATION OF BONDS AND STOCK

Objectives: After reading his chapter, you will
1. Understand the role of bonds in financial markets.
2. Distinguish between different types of bonds, such as zero-coupon, perpetual, discount, convertible, and junk bonds and apply the bond pricing formulas to evaluate these bonds.
3. Understand the concepts of equity capital, stock, and dividends.
4. Apply Gordon's growth model to evaluate the equity of a firm.
5. Find the value of a stock with supernormal growth for a few periods followed by normal growth.

3.1 Video 03A Capital formation

Corporations need *capital*, meaning money, to run their business. They need the money to make *capital investments*, which are investments in land, buildings, equipment, and machinery. In order to acquire capital the firms turn to *investors*. Figure 3.1 represents the relationship between the corporations and investors.

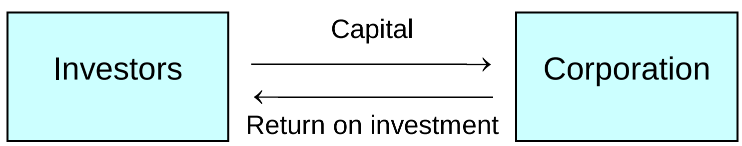


Fig. 3.1: The relationship between the investors and a corporation.

Examining the long-term capital structure of a company, we find that the capital comes in two forms: *debt* and *equity*. When a company acquires debt capital, it simply borrows money on a long-term basis from the investors. A company can also borrow money from a financial institution for the short-term. The firms issue financial instruments called *bonds* and sell them to the investors for cash. Bonds are merely promissory notes that promise to pay the investors the interest on the bonds regularly, and then pay the principal when the bonds mature.

When a corporation wants to raise equity capital, it sells *stock* to the investors. The stockholders then become part owners of the company. The ownership of stock gives them an equity interest in the company. There are important differences between debt and equity capital. For instance, the bonds mature after several years and the company must redeem the bonds by paying the principal back to the investors. There is no maturity date for the stock. The bondholders receive regular interest payments from the company. The stockholders may or may not receive dividends from the company. The stockholders vote for the election of board of directors, but the bondholders do not have any voting rights. The board of directors has the ability to make important decisions at the company, such as hiring or firing of its president.

3.2 Valuation of Bonds

The face amount of a typical bond is $1,000. The market value of the bond could be more than $1,000, and then it is selling at a *premium*. A bond with a market value less than $1,000 is selling at a *discount*, and a bond, which is priced at its face value, is selling at *par*. The market price of a bond is usually quoted as a percentage of its face value. For instance, a bond selling at 95 is really selling at 95% of its face value, or $950.

Figure 3.2 shows an advertisement that appeared in the *Wall Street Journal* of July 23, 1997. Dynex Capital, Inc. issued bonds with a total face value of $100 million in July 1997. The bonds carried a coupon of $7^{7}/_{8}$%. This means that each bond pays $78.75 in interest every year. Actually, half of this interest is paid every six months. The bonds will mature after 5 years, which is relatively short time for bonds. They are senior notes in the sense that the interest on these bonds will be paid ahead of some other junior notes. This makes the bonds relatively safer.

$100,000,000

DYNEX

Dynex Capital, Inc.

$7^{7}/_{8}$% Senior Notes Due July 15, 2002

Interest Payable January 15 and July 15

Price 99.900%

plus accrued interest from July 15, 1997

Paine Webber Incorporated Smith Barney Incorporated

Fig. 3.2: A bond advertisement in Wall Street Journal.

The price of these bonds is $999 for each $1,000 bond. Occasionally, the corporations may reduce the price of a bond and sell them at a discount from their face value. This is true if the coupon is less than the prevailing interest rates, or if the financial condition of the company is not too strong. The buyer must also pay the accrued interest on the bond. If an investor buys the bond on July 25, 1997, he must pay accrued interest for 10 days. When the bonds are publicly traded, they will be listed as "Dynex $7^{7}/_{8}$s02." The information about the bonds is frequently displayed as: Madison Company 4.75s33. We learn to interpret it as follows:

Madison Company: This is the name of the entity that issues the bonds

4.75: This is the coupon rate, or the annual rate of interest paid on the bonds, that is 4.75% per annum

s: This is just a separator between the two numbers

33: This is the year when the bond will mature, namely, 2033

The two companies listed at the bottom of the advertisement, Paine Webber Incorporated and Smith Barney Incorporated, are the *underwriters* for this issue. Underwriters, or investment banking firms, such as Merrill Lynch, will take a certain commission for selling the entire issue to the public.

Since the appearance of this advertisement, several changes have occurred. On November 3, 2000, Paine Webber merged with UBS AG, a Swiss banking conglomerate. Smith Barney is now part of Morgan Stanley Smith Barney. Corporations no longer use fractions in identifying the coupon rates, instead decimals are used universally.

An important feature of every bond issue is the indenture. The indenture is a detailed legal contract between the bondholders and the corporation that spells out the rights and obligations of both parties. In particular, it gives the bondholders the right to sue the company and force it into bankruptcy, if the company fails to pay the interest payments on time. This provides safety to the bondholders, and puts serious responsibility on the corporation.

The two factors that determine the interest rate of a bond are the creditworthiness of the corporation and the prevailing interest rates in the market. A company that is doing well financially, and has good prospects in the future, will have to pay a lower rate of interest to sell bonds. A company that is close to bankruptcy will have a hard time selling its bonds, and must attach a high coupon rate to attract the investors.

The term sinking fund describes the amount of money that a company puts aside to retire its bonds. For example, a company issues bonds with face value $50 million, which will mature in 20 years. During the last five years of their existence, the company may set aside $10 million per year to buy back, or retire their bonds. This $10 million is the sinking fund payment. This procedure spreads the loan repayment over a five-year period and is easier for the company to manage.

To retire the bonds, the corporation may buy the bonds in open market if they are selling below par. The corporation may also *call* the bonds, depending on the provisions of the indenture, by paying the more than the face value of the bonds to the bondholders. Such bonds are called callable bonds.

We can evaluate a bond by finding the present value of the interest payments and that of the principal. The proper discount rate that calculates the present value depends on the risk of the bonds. The risky bonds have a relatively higher discount rate. Further, the discount rate is also the rate of return required by an investor buying that bond. The basic financial principle is:

> The present value of a bond is simply the present value of all future cash flows from the bond, properly discounted.

We may express the above statement as follows

$$B = \sum_{i=1}^{n} \frac{C}{(1+r)^i} + \frac{F}{(1+r)^n} \tag{3.1}$$

The first term on the right side is the present value of the coupon payments, or the interest payments in dollars. The second term is the present value of the face amount of the bond in dollars. This resembles equation (2.9).

Recently, CalTech issued very long maturity bonds, 100 years to be exact. In 1997, a company in Luxembourg issued bonds that would mature after 1,000 years. British Government has issued perpetual bonds, called consols, which are still available today and carry a coupon rate of 2½%. In principle, an American company can issue perpetual bonds that will never mature but the Federal Government prohibits that.

Perpetual bonds have an infinite life span. In essence, they are perpetuities. The bondholders continue to receive interest payments and if they want to, they can always sell the bonds to other investors. Since the bond is never going to mature, the implicit assumption is that the investors will never receive the face amount of such a bond. From (2.7), when n approaches infinity, the summation becomes C/r. The second term for the present value of the face amount also approaches zero. From (3.1) we get the simple formula for perpetual bonds.

$$B = \frac{C}{r} \tag{3.2}$$

Some companies try to conserve cash and they may sell zero-coupon bonds. These bonds make no periodic interest payments and they pay the entire accumulated interest and the principal at the maturity of the bond. Because of this feature, these bonds sell at a substantial discount from their face value. For instance, General Motors issued zero coupon bonds in 1996 due to mature in 2036. In January 2007, these bonds were selling at 38.11, or \$381.10 per \$1000 bond. For zero-coupon bonds, the first term in (3.1) is zero because C is zero. This leaves only the second term for the valuation of zero-coupon bonds as follows:

$$B = \frac{F}{(1+r)^n} \tag{3.3}$$

Occasionally, a company may issue convertible bonds. A bondholder, at his discretion, can exchange a convertible bond for a fixed number of shares of stock of the corporation. For example, the bondholder may get 50 shares of stock by giving up the bond. If the price of the stock is \$10 a share, then the conversion value of the bond will be \$500, that is, the bond can be converted into \$500 worth of stock. The market value of the bond will always be more than the conversion value. If the price per share rises to \$25, then the price of the bond will be at least 50(25) = \$1250. Thus, the convertible bonds are occasionally trading above their face value.

At times, the financial health of a company deteriorates quite a bit. The company may even stop paying interest on the bonds, and there is little hope of recovery of principal of these bonds. Such bonds, with extremely high investment risk, are frequently labeled as "junk" bonds.

An investor buys a bond for its rate of return, or its *yield*. We define the current yield, y, of a bond as follows.

$$y = \frac{\text{Annual interest payment of the bond}}{\text{Current market value of the bond}}$$

The annual interest payment of the bond equals cF, where c is the coupon rate, and F is the face value of the bond. With B being the market value of the bond, we may write

$$y = cF/B \tag{3.4}$$

This represents the return on the investment provided the bond is held for a short period.

Holding a bond to maturity, one receives money in the form of interest payments, plus there is a change in the value of the bond. The annual interest payment of the bond is cF, as seen before. If you have bought the bond at a discount, it will rise in value reaching its face value at maturity. Or, the bond may drop in price if it has been bought at a premium. In any case, it should be selling for its face value at maturity. The total price change for the bond is $F - B$, where F is the face value of a bond and B is its purchase price. This change may be positive or negative depending upon whether F is more, or less, than B. On the average, the price change per year is $(F - B)/n$, where n is the number of years until maturity. On the average, the price of the bond for the holding period is $(F + B)/2$. Thus the yield Y, of a bond is given, approximately, by dividing the annual return by the average price. This is given by:

$$Y \approx \frac{\text{annual interest payment + annual price change}}{\text{average price of the bond for the entire holding period}}$$

Or,

$$Y \approx \frac{cF + (F - B)/n}{(F + B)/2} \tag{3.5}$$

Let us define b as the market value of the bond expressed as a fraction of its face value. For instance, if a bond is selling at 90% of its face face, or \$900 per \$1000 bond, then b = .9. With this definaition, it is possible to write (3.5) as

$$Y \approx \frac{2(cn + 1 - b)}{n(1 + b)} \tag{3.5a}$$

For a bond selling at par, $b = 1$, meaning the bond is selling at its face value. In that case, (3.5a) gives $Y = c$.

In equation (3.1), the discount rate r is also equal to the yield to maturity, Y. The reason for the approximation in the equation (3.5) is that the value of a bond does not reach the face amount linearly with time, as seen in Figure 3.3.

Consider a bond that has 8% coupon, pays interest semiannually, and will mature after 10 years. Assume that the investors require 10% return on these bonds. Then the current value of the bond is

$$B = \sum_{i=1}^{20} \frac{40}{1.05^i} + \frac{1000}{1.05^{20}} = \$875.38$$

As the bond approaches maturity, its value reaches \$1,000. This is shown in Fig. 3.3. Notice that the curve is not a straight line. The bond value rises slowly at first and then more rapidly when it is close to maturity.

Equation (3.5) calculates the yield to maturity of a bond only approximately. To find it more accurately, we depend on the alternate definition of yield to maturity: *The yield to maturity of a bond is that particular discount rate, which makes the present value of the cash flows to be equal to the market value of the bond*. Thus, we go back to (3.1), put known values of *B*, *n*, *C*, and *F*, and evaluate the unknown *r*. That is the yield to maturity of the bond. We need a set of Maple or WolframAlpha instructions to get the final answer.

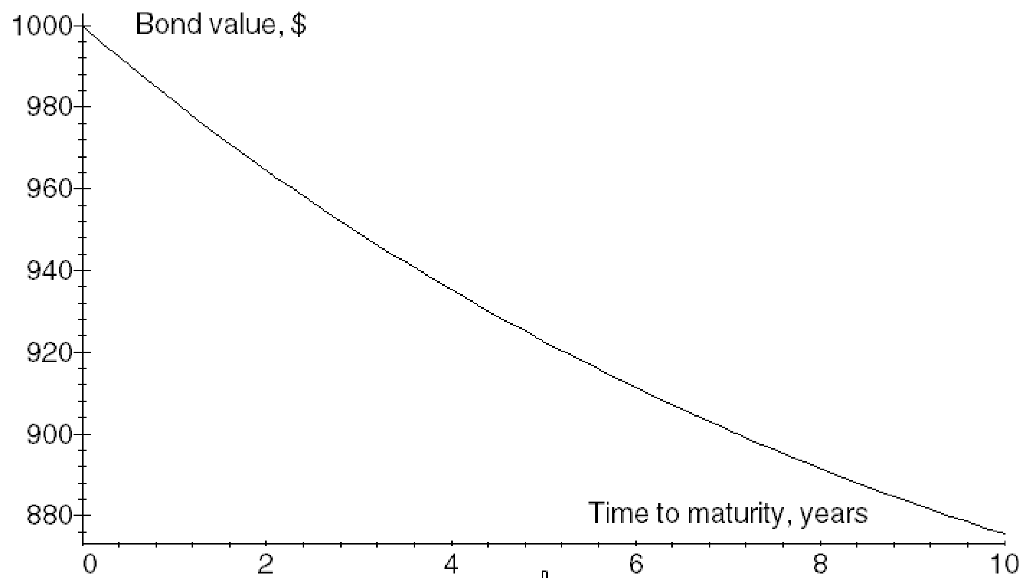


Fig. 3.3: The value of a bond with respect to time to maturity. Face value \$1000, coupon 8%, 10 years to maturity, semiannual payments, yield to maturity 10%.

The US Government borrows heavily in the financial markets by issuing Treasury bonds. They are issued with maturity date ranging from six months to thirty years. The yield of these bonds fluctuates. The following table gives the yield of Treasury securities on January 5, 2007.

US Treasury Bond Rates, January 5, 2007				
Maturity	**Yield**	**Yesterday**	**Last Week**	**Last Month**
3 Month	4.88	4.87	4.84	4.83
6 Month	4.87	4.85	4.82	4.83
2 Year	4.72	4.67	4.78	4.56
3 Year	4.65	4.60	4.71	4.47
5 Year	4.62	4.57	4.65	4.43
10 Year	4.63	4.58	4.68	4.47
30 Year	4.72	4.69	4.79	4.58

Table 3.1: Source: http://finance.yahoo.com/bonds

One can plot the yield against the time to maturity to get the Treasury yield curve, shown in Figure 3.4. The curve is plotted on a semilog scale to accommodate long maturity dates. Normally, one expects that the longer maturity bonds have a higher yield, but this is not the case in January 2007. Hence we see an inverted yield curve in the diagram.

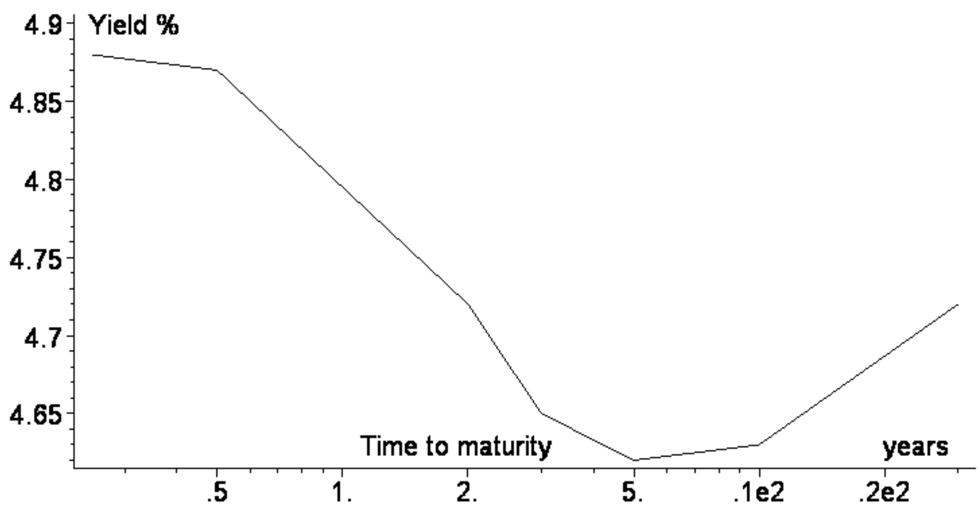


Figure 3.4. The inverted Treasury yield curve on January 5, 2007. On the *x*-axis, .1e2 means 10 years, and .2e2 is 20 years.

Corporate Bonds, January 5, 2007				
Maturity	**Yield**	**Yesterday**	**Last Week**	Last Month
2yr AA	5.04	4.98	5.11	4.86
2yr A	5.13	5.08	5.20	4.92
5yr AAA	5.06	5.03	5.11	5.19
5yr AA	5.13	5.09	5.17	4.93
5yr A	5.20	5.16	5.23	4.99
10yr AAA	5.18	5.07	5.30	5.08
10yr AA	5.32	5.33	5.42	5.19
10yr A	5.43	5.37	5.47	5.26
20yr AAA	5.68	5.71	5.76	5.06
20yr AA	5.76	5.79	5.84	5.68
20yr A	5.82	5.85	5.90	5.71

Table 3.2: The yield of bonds as a function of quality and time to maturity. Source: http://finance.yahoo.com/bonds January 5, 2007

Table 3.2 shows the yields of corporate bonds on January 5, 2007. The letters AAA, AA, and A represent the quality of bonds, or bond rating, by Fitch. The least risky bonds are designated by AAA, and so on. We notice two things. First, the longer maturity bonds of the same quality rating have a higher yield. For instance, for bonds with A rating, the yield for 2- year maturity is 5.13%; and for 20 years, it is 5.82%. Second, the yield is higher for riskier bonds. Consider 5-year bonds. The yield rises from 5.06% to 5.20% when the rating drops from AAA to A.

Issue	Price	Coupon %	Maturity date	YTM %	Current Yield	Fitch Ratings	Callable
Federal Home Ln Mtg	99.00	5.000	27-Jan-2017	5.128	5.051	AAA	Yes
Goldman Sachs	104.40	5.750	1-Oct-2016	5.168	5.508	AA	No
Emerson Electric	100.53	5.125	1-Dec-2016	5.056	5.098	A	No
Clear Channel Comm.	90.90	7.250	15-Oct-2027	8.165	7.976	BBB	No
Scotia Pacific	81.50	7.710	20-Jan-2014	11.634	9.460	BB	No
Brookstone	99.88	12.000	15-Oct-2012	12.020	12.015	B	Yes
Fedders No Am	72.50	9.875	1-Mar-2014	16.575	13.621	CCC	Yes
Wise Metals	90.74	10.250	15-May-2012	12.678	11.296	CC	Yes

Table 3.4: The yield of bonds as a function of quality and time to maturity. Source: Source: http://finance.yahoo.com/bonds January 5, 2007

Table 3.4 shows a sampling of bonds available in the market in January 2007. They are arranged in terms of their quality rating, the least risky bonds are the top and the riskiest ones at the bottom.

Normally, when a buyer buys a bond he has to pay the accrued interest on the bond. This is the interest earned by the bond since the last interest payment date. Occasionally some bonds trade without the accrued interest and they are thus *dealt in flat*. Some corporations gradually get deeper in financial trouble. As they come closer to bankruptcy, their bonds lose their value drastically. Finally, they become *junk* bonds.

Video 03B Examples

3.1. An investor wants to buy a bond with face value $1,000 and coupon rate 12%. It pays interest semiannually and it will mature after 5 years. If her required rate of return is 18%, how much should she pay for the bond?

The present value of a bond is the sum of the present value of its interest payments plus the present value of its face value. The annual interest on the bonds is .12(1000) = $120, and thus the semiannual interest payment is $60. The annual required rate is 18%, or 9% semiannually. This is the discount rate. There are 10 semiannual periods in 5 years. Put $n = 10$, $r = .09$, $F = 1000$ in (3.1), which gives

$$B = \sum_{i=1}^{10} \frac{60}{1.09^i} + \frac{1000}{1.09^{10}} = \frac{60(1 - 1.09^{-10})}{0.09} + \frac{1000}{1.09^{10}} = \$807.47$$

She should pay $807.47 for the bond. ♥

To verify the answer at WolframAlpha, use the following instruction.

WRA **`sum(60/1.09^i,i=1..10)+1000/1.09^10`**

3.2. American Airlines bonds pay interest on January 15 and July 15, and they will mature on July 15, 2017. Their coupon rate is 11%. Because of the risk characteristics of American Airlines, you require a return of 15% annually on these bonds. How much should you pay for a $1,000 bond on January 16, 2011?

The bonds will mature in 6.5 years and you will receive 13 interest payments of $55 each. obtained by setting n = 13, r = .075, F = 1000 in (3.1), which gives the PV of these interest payments, plus the discounted face value as

$$B = \sum_{i=1}^{13} \frac{55}{1.075^i} + \frac{1000}{1.075^{13}} = \frac{55[1 - 1.075^{-13}]}{0.075} + \frac{1000}{1.075^{13}} = \$837.48 ♥$$

3.3. A zero coupon bond with face value $1,000 and 6.25 years until maturity is available in the market. Because of its risk characteristics, you require a 11.5% return, compounded annually, on this bond. How much should you pay for it?

For a zero-coupon bond, use

$$B = \frac{F}{(1 + r)^n} \tag{3.3}$$

Put F = 1000, r = .115, n = 6.25,

to get

$$B = \frac{1000}{1.115^{6.25}} = \$506.44 ♥$$

3.4. Canopus Corporation's 9% coupon bonds pay interest semiannually, and they will mature in 10 years. You pay 30% tax on interest income, but only 15% on capital gains. Your after-tax required rate of return is 12%. Assume that you pay taxes once a year. What is the maximum price you are willing to pay for a $1,000 Canopus bond?

Suppose you pay x dollars for a $1,000 bond. The annual interest is $90; or $45 every six months. For semiannual cash flows, the discount rate is 6%, which is one-half of the annual required rate of return. In ten years, you will get 20 semiannual payments.

The annual tax on $90 interest income is .3(90) = $27.

After 10 years, you receive the face value of bond, $1,000, and you have a capital gain of $(1000 – x)$. However, you have to pay tax on the capital gain, which comes to $(.15)(1000 – x) = 150 – .15x$. The after-tax amount is thus $1000 – (150 – .15x) = 850 + .15x$.

Apply the financial principle:

PV of the bond	=	PV of 20 semiannual interest payments, discounted at 6%	–	PV of 30% of $90, paid in taxes annually for 10 years	+	PV of the after-tax final payment, which is $1000 minus 15% of the difference between 1000 and x

Write it in symbols,

$$x = \sum_{i=1}^{20} \frac{45}{1.06^i} - \sum_{i=1}^{10} \frac{27}{1.12^i} + \frac{850 + .15x}{1.12^{10}} \quad \text{(A)}$$

Move the terms with x on the left side of the equation and rewrite it as

$$x\left(1 - \frac{.15}{1.12^{10}}\right) = \sum_{i=1}^{20} \frac{45}{1.06^i} - \sum_{i=1}^{10} \frac{27}{1.12^i} + \frac{850}{1.12^{10}}$$

Use (3.1) to complete the summation as

$$x\left(1 - \frac{.15}{1.12^{10}}\right) = \frac{45(1 - 1.06^{-20})}{.06} - \frac{27(1 - 1.12^{-10})}{.12} + \frac{850}{1.12^{10}}$$

Or, $.9517040145\, x = 516.1464548 - 152.5560218 + 273.6772511$

Or, $x = 669.6070148 = \$669.61$ ♥

To verify the answer at WolframAlpha, use the following instruction to solve (A).

WRA `x=sum(45/1.06^i,i=1..20)-sum(.3*90/1.12^i,i=1..10)+(1000-(1000-x)*.15)/1.12^10`

3.5. You have bought a \$1,000 bond for \$450, with a coupon of 5%, which has 10 years until maturity. The interest is paid semiannually. Find the yield to maturity for this bond.

Here we use

$$Y \approx \frac{cF + (F - B)/n}{(F + B)/2} \tag{3.5}$$

Put $c = .05$, $F = 1000$, $B = 450$, $n = 10$, in (3.5), which gives

$$Y \approx \frac{.05*1000 + (1000 - 450)/10}{(1000 + 450)/2} = .1448$$

Or, about 14.5% per year. ♥

To find the yield accurately, we set the current price equal to the sum of discounted future interest payments and the face value. Suppose the unknown yield to maturity is r, which is also the proper discount rate to use in the bond valuation equation (3.1). Assume the bond pays interest semiannually. Therefore, we should use $r/2$ as the discount rate for \$25 semiannual interest payments.

$$450 = \sum_{i=1}^{20} \frac{25}{(1 + r/2)^i} + \frac{1000}{(1 + r/2)^{20}}$$

We may solve this equation by using Maple. If we enter the command

```
fsolve(450=sum(25/(1+r)^i,i=1..20)+1000/(1+r)^20,r);
```

the answer shows up as .1635007175 or about 16.35%.

To solve the problem using WolframAlpha, write the above equation as

WRA `450=25*(1-1/(1+r/2)^20)/r*2+1000/(1+r/2)^20`

You can do it on Excel by setting up the spreadsheet as follows. Adjust the value in cell B5 until the value in cell C5 becomes close to zero. The exact yield to maturity is in cell B5, with the quantity in cell C5 equal to −.002, which is less than a penny.

	A	B	C
1	Face amount of bond =	1000	dollars
2	Market value of bond =	450	dollars
3	Coupon rate =	5%	
4	Time to maturity =	10	years
5	Yield to maturity =	16.35%	=B2-B3*B1/2*(1-1/(1+B5/2)^(2*B4))/B5*2-B1/(1+B5/2)^(2*B4)

This is the annual return, or 16.35%. This is the exact answer with four digit accuracy.

Video 03C **3.6.** Bareilly Corporation bonds will mature after 3 years, and carry a coupon rate of 12%. They pay interest semiannually. However, due to poor financial condition of the company, you believe that there is a 30% probability Bareilly will go bankrupt in any given year. In case of bankruptcy, you expect that the company will make the interest payments for that year, and also pay only 20% of the principal at the end of that year. If your required rate of return is 12%, find the value of this bond.

Organize the calculation as follows.

Event	Probability	PV of cash flows	Prob*PV
Bankrupt in first year	.3	$\sum_{i=1}^{2}\frac{60}{1.06^i}+\frac{200}{1.06^2}=288.00$	.3(288.00)
Bankrupt in second year	.7(.3) = .21	$\sum_{i=1}^{4}\frac{60}{1.06^i}+\frac{200}{1.06^4}=366.33$	.21(366.33)
Bankrupt in third year	.7(.7)(.3) = .147	$\sum_{i=1}^{6}\frac{60}{1.06^i}+\frac{200}{1.06^6}=436.03$	.147(436.03)
Survive all 3 years	.7(.7)(.7) = .343	$\sum_{i=1}^{6}\frac{60}{1.06^i}+\frac{1000}{1.06^6}=1000$	.343(1000)
Sum of the above	1		$570.43

Note that the sum of the probabilities is 1 or 100%. The value of the bond is $570.43 ♥

3.7. Compton Company bonds pay interest semiannually, and they will mature after 10 years. Their current yield is 8%, whereas their yield to maturity is 10%. Find the coupon rate and the market value of these bonds. Hint: use (3.1) and (3.4).

Since we have to find the value of two unknown quantities, the coupon rate, c and the market value of the bond, B, we need to develop two equations. Recall that the yield to maturity of a bond is the same as the required rate of return r. Assume semiannual interest payments.

Semiannual required rate of return or the discount factor, $r = .05$,
The number of interest payments of the bond, $n = 20$,
The face value of the bond, $F = 1000$,
The dollar value of each interest payment, $C = cF/2 = 500c$, in (3.1)

$$B = \sum_{i=1}^{20} \frac{500c}{1.05^i} + \frac{1000}{1.05^{20}} = \frac{500c(1 - 1.05^{-20})}{.05} + \frac{1000}{1.05^{20}}$$

Or, with some simplification $B = 6231.105171c + 376.8894829$ (1)

This is the first equation. To get the second equation, put $y = .08$ in (3.4). This gives

$$.08 = c(1000)/B \qquad (2)$$

Solving (2) for c, we find $c = (.08/1000)B$

Substituting the above value of c in (1), we get

$$B = 6231.105171(.08/1000)B + 376.8894829$$

Or, $B = .4984884137B + 376.8894829$

Or, $B(1 - .4984884137) = 376.8894829$

Or, $B = 376.8894829/(1 - .4984884137) = 751.5070303 \approx \$751.51$

Going back to (2), $c = (.08/1000)\ 751.5070303 = .06012056242 \approx 6.012\%$

This gives the coupon rate as 6.012%, and the market value of the bond to be \$751.51. ♥

To solve the problem using WolframAlpha, write the two basic equations as

WRA `B=sum(500*c/1.05^i,i=1..20)+1000/1.05^20,.08=c*1000/B`

Money Market Rates

Tuesday, July 31, 2007, Wall Street Journal

PRIME RATE: 8.25% (effective 6/29/06). This is a benchmark rate used by banks in making loans to their commercial customers. The best customers pay close to the prime rate, less creditworthy customers pay more.

DISCOUNT RATE: 6.25% (Primary) (effective 6/29/06). This is the rate charged by the Federal Reserve for the loans made to the member banks.

FEDERAL FUNDS: 5.4375%, high, 4.500% low, 4.250% near closing bid, 5.00% offered. Effective rate 5.32%. Reserves traded by the member banks for overnight use in amounts of $1 million or more.

CALL MONEY: 7.00% (effective 6/29/06). This is the rate of interest used by stockbrokers for making loans to their customers for the purchase of common stocks.

COMMERCIAL PAPER: placed directly by General Electric Capital Corporation, 5.24% 30 to 44 days, 5.25% 45 to 61 days, 5.28% 62 to 89 days, 5.29% 90 to 119 days, 5.30% 120 to 190 days, 5.29% 191 to 219 days, 5.28% 220 to 249 days, 5.27% 250 to 270 days.

CERTIFICATES OF DEPOSIT: 5.28% one month, 5.36% three months, 5.46% six months.

LONDON INTERBANK OFFERED RATE (LIBOR): 5.3300% one month, 5.42625% three months, 5.5150% six months, 5.5675% one year. The rate is set by the British Banker's Association, and is used by one bank making loan to another bank. This is a key rate used in international transactions, especially interest rate swaps.

FOREIGN PRIME RATES: Canada 6.25%, European Central Bank 4.00%, Japan 1.875%, Switzerland 4.42%, Britain 5.75%, Australia 6.25%, Hong Kong 8.00%.

TREASURY BILLS: Results of the Monday, August 31, 2007, auction of the short-term T-bills sold at a discount in units of $1,000 to $1 million. 5.055% 4 weeks, 4.825% 13 weeks, 4.800% 26 weeks.

MERRILL LYNCH READY ASSETS TRUST: 4.70%, average rate of return, after expenses, for the past 30 days; not a forecast of future returns.

CONSUMER PRICE INDEX: June 208.4, up 2.7% from a year ago. Bureau of Labor Statistics.

Table 3.3: Money rates

3.3 Valuation of Stock

The two principal components of the capital structure of a company are its equity and debt. A corporation sells its stock to the investors to raise equity capital. The financial markets ultimately determine the value of a share of stock. If the company is in strong financial condition and it has good earnings prospects, then the investors will aggressively buy its stock and raise the price per share. The market value of a stock could be quite different from its *book value* or its accounting value. The value of the stock depends upon the expectations of the investors regarding the future earnings and growth possibilities of the firm.

Table 3.5 gives the information about the stocks of some well-known companies. The information is for close of business on January 5, 2007. The first column shows the range of the stock price, in dollars, for the past 52 weeks. The next two columns give the name of the company and its stock symbol. The fourth and fifth columns show the closing price of the stock and its net change. General Electric, for instance, closed at $37.56 per share, down 19¢. The next two columns show the annual dividend per share and the dividend yield. For Boeing, the annual dividend is $1.40 per share and its dividend yield is 1.40/89.15 = .0157 = 1.57%.

52 Week Range	Stock	Sym-bol	Close	Net Chg	Div	Yld %	PE	Volume 1000s	Market Cap	β
65.90-92.05	Boeing	BA	89.15	-0.38	1.40	1.57	41.50	3,168	70.4B	0.62
44.81-57.00	Citigroup	C	54.77	-0.29	1.96	3.60	11.79	13,130	269.1B	0.44
32.06-38.49	Gen Electric	GE	37.56	-0.19	1.12	3.00	22.83	26,729	387.2B	0.51
32.85-43.95	Home Depot	HD	37.79	-0.78	0.90	2.30	13.60	21,676	81.21B	1.28
21.46-30.26	Microsoft	MSFT	29.64	-0.17	0.40	1.30	23.69	44,680	291.4B	0.71
27.83-37.34	PP&L	PPL	35.55	-0.63	1.10	3.10	15.74	1,048	13.56B	0.24

Table 3.5: Stock prices. Source: Finance.Yahoo.com, January 7, 2007

The next column shows the P-E ratio, which is the ratio between the price of the stock per share and the earnings per share. For Citigroup, it is 11.79. This gives the earnings per share as 54.77/11.79 = $4.65 per share. Citigroup pays $1.96 as dividends out of this money. Thus its dividend payout ratio is 1.96/4.65 = 42.15%. The next column shows the trading volume. More than 44 million shares of Microsoft changed hands that day. The next column shows the total market value of the company, in billions of dollars. The last column shows the β of the stock. Beta is a measure of the risk of the stock. We shall look at it more closely in chapter 5. It is interesting to note that all stocks went down on this trading day, but it is not surprising because all the major market indicators shown in Table 3.6 also went down.

Symbol	Last	Change
Dow	12,398.01	↓82.68 (0.66%)
NASDAQ	2,434.25	↓19.18 (0.78%)
S&P 500	1,409.71	↓8.63 (0.61%)

Table 3.6. The stock market indices on January 5, 2007, source, Finance.Yahoo.com

An investor buying the common stock of a corporation is looking at two possible returns: the receipt of cash dividends and the growth of the company. Let us make a couple of simplifying assumptions to develop a formula for stock valuation.

1. Assume that the firm is growing steadily, that is, its growth rate remains constant. This also means that the dividends of the firm are also growing at a constant rate. In reality, the firms grow in an uncertain way.

2. The growth is supposed to continue forever. This is also quite unrealistic because the companies tend to grow rapidly at first, then the growth rate slows down, and some mature firms actually decline in value.

Even though the assumptions are not very good, its gives a fairly accurate result. Let us define:

P_0 = price of the stock now
D_1, D_2, D_3, ... = the stream of cash dividends received in year 1, 2, 3, ...
g = growth rate of the dividends
R = the required rate of return by the stockholders

Assuming that the company is going to grow forever, then the price of the stock now is just the discounted value of all future dividends.

$$P_0 = \frac{D_1}{1+R} + \frac{D_2}{(1+R)^2} + \frac{D_3}{(1+R)^3} + \ldots \infty$$

But $D_2 = D_1(1+g)$, $D_3 = D_1(1+g)^2$, $D_4 = D_1(1+g)^3$, and so on. Thus

$$P_0 = \frac{D_1}{1+R} + \frac{D_1(1+g)}{(1+R)^2} + \frac{D_1(1+g)^2}{(1+R)^3} + \ldots \infty$$

This becomes an infinite geometric series, with the first term $a = \frac{D_1}{1+R}$ and the ratio of terms $x = \frac{1+g}{1+R}$. Using equation (1.5),

$$S_\infty = \frac{a}{1-x} \tag{1.5}$$

we get

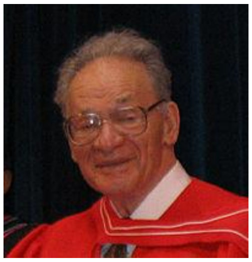
$$P_0 = \frac{D_1}{(1+R)\left(1 - \frac{1+g}{1+R}\right)}$$

Simplifying it,

$$P_0 = \frac{D_1}{R-g} \tag{3.6}$$

To solve the problem using WolframAlpha, write the above equations as

WRA **sum(D1*(1+g)^(i-1)/(1+R)^i,i=1..infinity)**


Myron J. Gordon (1920-2010)

Equation (3.6) gives us the valuation of a common stock. It is a well-known result called Gordon's growth model, named after Myron Gordon. This result is valid only if $R > g$. Although it is based on some unrealistic assumptions, it does provide fairly accurate stock valuation. The following problems will illustrate the general method of evaluation of equity when the growth rate is not constant.

3.4. Preferred Stock

Preferred stock is similar to the common stock. It is part of the equity-portion of the capital structure of the firm. However, the preferred stock has several important differences.

First, the dividends of the preferred stock remain constant. That is, the rate of growth of dividends is zero. If a preferred stock pays $3 annually, it will continue to do so, even if the issuing company experiences very strong growth. In this respect, the preferred stock is similar to a bond, which pays constant interest payments.

Second, the dividends of the preferred stock are paid ahead of the common stock, hence the name *preferred stock*. This makes the preferred stock a safer investment relative to the common stock, but it is not quite as safe as the bonds of the same corporation.

Third, the preferred stock usually has a limited life. This means that a company may issue a preferred stock and then take it out of circulation after five years. The preferred stockholders must give up their stock and receive cash. In this respect, preferred stock is similar to a bond, which will also mature after five or ten years.

Fourth, a preferred stock has no voting rights. The common stockholders have the right to elect the board of directors of a firm, which makes important decisions at the firm. The bondholders of a firm do not have voting rights either.

To summarize, the preferred stock has some features of a bond and some other features of a stock. It is a hybrid security.

Examples

Video 03E **3.8.** The common stock of Steerforth Inc has just paid a dividend of $2.00. The dividends are expected to grow at the rate of 10% for the next three years, and then at the rate of 5% forever. Find the price of this stock, assuming the required rate of return is 20%.

The price of a stock is equal to the sum of discounted future dividends received by an investor. The current dividend is $2, but it will grow at the rate of 10% for the next three years. The dividend after one year is 2(1.1); after two years, it becomes $2(1.1)^2$; and after three years, it is $2(1.1)^3$. After that, the growth slows down to 5%. Thereafter the dividends are: $2(1.1)^3(1.05)$ after 4 years, and $2(1.1)^3(1.05)^2$ after 5 years, and so on. All these numbers are discounted at the rate of 20%. If P_0 is the current price of the stock, then

$$P_0 = \frac{2\,(1.1)}{1.2} + \frac{2\,(1.1)^2}{1.2^2} + \frac{2\,(1.1)^3}{1.2^3} + \frac{2\,(1.1)^3\,(1.05)}{1.2^4} + \frac{2\,(1.1)^3\,(1.05)^2}{1.2^5} + \ldots$$

Starting with the third term, $\frac{2\,(1.1)^3}{1.2^3}$, it becomes an infinite series with $a = 2(1.1/1.2)^3$, and $x = 1.05/1.2$. The sum of all terms is thus

$$P_0 = \frac{2\,(1.1)}{1.2} + \frac{2\,(1.1)^2}{1.2^2} + \frac{2\,(1.1/1.2)^3}{1 - 1.05/1.2} = \$15.84 \ ♥$$

To solve the problem using WolframAlpha, write the above equations as

WRA `2*1.1/1.2+2*(1.1/1.2)^2+sum(2*(1.1/1.2)^3*(1.05/1.2)^i,i=0..infinity)`

3.9. Sirius Inc. common stock just paid a quarterly dividend of \$1.00. Investors expect this dividend to grow at annual rate of 4%, compounded quarterly, for the next 10 quarters. Then it will remain constant in future. The stockholders require a return of 12% on their investment in Sirius. What is the current market price of Sirius common stock?

The growth rate of dividends is 4% per year, or 1% per quarter. The required rate of return per year is 12%, or 3% per quarter. Adding together the discounted value of the return from the first 10 quarters and the infinite many quarters thereafter will give the value of the stock.

For the first 10 quarters, the dividends are growing at 1% per quarter and they are $D_1 = 1.01$, $D_2 = 1.01^2$, $D_3 = 1.01^3$, and so on. For the remaining quarters, starting with the 11th quarter, the dividend will remain constant at 1.01^{10}. The discount rate is 3%. Thus

$$P_0 = \frac{1.01}{1.03} + \frac{1.01^2}{1.03^2} + \frac{1.01^3}{1.03^3} + \ldots + \frac{1.01^9}{1.03^9} + \frac{1.01^{10}}{1.03^{10}} + \frac{1.01^{10}}{1.03^{11}} + \frac{1.01^{10}}{1.03^{12}} + \frac{1.01^{10}}{1.03^{13}} + \ldots \infty$$

Write the first ten terms as the summation of one geometric series and the remaining terms as another infinite geometric series. For the first series, we let $a = 1.01/1.03$, $x = 1.01/1.03$, and $n = 10$ in equation (1.4). For the remaining terms, put $a = 1.01^{10}/1.03^{11}$, and $x = 1/1.03$ in (1.5). This gives

$$P_0 = \frac{(1.01/1.03)[1 - (1.01/1.03)^{10}]}{1 - 1.01/1.03} + \frac{1.01^{10}/1.03^{11}}{1 - 1/1.03}$$

After simplification, the stock price comes out to be \$36.39. ♥

To solve the problem using WolframAlpha, write the above equation as

WRA `sum((1.01/1.03)^i,i=1..10)+sum(1.01^10/1.03^i,i=11..infinity)`

3.10. Gabon Corporation is expected to have the following growth rates: 10% during the first three years, 5% during the next three years, and then zero forever thereafter. Gabon just paid its annual dividend of $5. What is the price of a share of Gabon stock if the stockholders require a return of 10% on their investment?

We can find the stock price by the following summation. The PV of cash flows for the first three years, next three years, and the remaining years, are shaded in different colors.

$$P_0 = \frac{5(1.1)}{1.1} + \frac{5(1.1)^2}{1.1^2} + \frac{5(1.1)^3}{1.1^3} + \frac{5(1.1)^3(1.05)}{1.1^4} + \frac{5(1.1)^3(1.05)^2}{1.1^5} + \frac{5(1.1)^3(1.05)^3}{1.1^6}$$

$$+ \frac{5(1.1)^3(1.05)^3}{1.1^7} + \frac{5(1.1)^3(1.05)^3}{1.1^8} + \frac{5(1.1)^3(1.05)^3}{1.1^9} + \ldots \infty$$

Canceling terms,

$$P_0 = 5 + 5 + 5 + \frac{5(1.05)}{1.1} + \frac{5(1.05)^2}{1.1^2} + \frac{5(1.05)^3}{1.1^3} + \frac{5(1.05)^3}{1.1^4} + \frac{5(1.05)^3}{1.1^5} + \ldots \infty$$

The shaded series is an infinite series. Add all items to get

$$P_0 = 15 + 5(1.05/1.1) + 5(1.05/1.1)^2 + \frac{5(1.05/1.1)^3}{1 - 1/1.1} = \$72.16 ♥$$

3.11. Troy Company has issued $5 cumulative preferred stock. The cumulative feature means that if the company is unable to pay the dividends in any year, it must pay them cumulatively next year. The probability that the company will actually pay the dividends in any year is 70%. At the end of three years, Troy *must* pay all the dividends and buy back the stock for $50 per share. If your required rate of return is 12%, how much should you pay for a share of this stock?

The company will certainly make the final payment in year 3. Consider the three-year period and the cash payments. There are four possible outcomes.

Year 1	Year 2	Year 3	Probability
$0	$0	$65	.3(.3) = 0.09
$0	$10	$55	.3(.7) = 0.21
$5	$0	$60	.7(.3) = 0.21
$5	$5	$55	.7(.7) = 0.49

The total probability of all outcomes is 1. The PV of the cash flows is thus

$$PV = 0.09\left(\frac{65}{1.12^3}\right) + 0.21\left(\frac{10}{1.12^2} + \frac{55}{1.12^3}\right) + 0.21\left(\frac{5}{1.12} + \frac{60}{1.12^3}\right) + 0.49\left(\frac{5}{1.12} + \frac{5}{1.12^2} + \frac{55}{1.12^3}\right)$$

$$= \$47.29$$

You should pay at most $47.29 for one share of stock. ♥

3.12. Mayfield Corporation stock is expected to pay a dividend of $2.00 one year from now, $2.50 two years from now, $3.00 three years from now, and then $4.00 a year at the end of the fourth and subsequent years. The stockholders of Mayfield require 15% return on their investment. Find the price of a share of Mayfield stock now, and just after the payment of the first dividend.

The value of the stock is the present value of all future dividends, discounted at the rate of 15%. Display the cash flows and their present values in the following table.

Year	1	2	3	4	5	6	...	∞
Dividend	$2.00	$2.50	$3.00	$4.00	$4.00	$4.00	...	$4.00
Discounted value	$\frac{2}{1.15}$	$\frac{2.5}{1.15^2}$	$\frac{3}{1.15^3}$	$\frac{4}{1.15^4}$	$\frac{4}{1.15^5}$	$\frac{4}{1.15^6}$	...	$\frac{4}{1.15^\infty}$

Discounting at the rate of 15%, write the present value of the cash flows as

$$P_0 = \frac{2}{1.15} + \frac{2.5}{1.15^2} + \frac{3}{1.15^3} + \frac{4}{1.15^4} + \frac{4}{1.15^5} + \frac{4}{1.15^6} + \ldots\ \infty$$

Look at the shaded terms. Note that starting with the fourth term, with $4 dividend, it becomes an infinite geometric series. The first term in the series, $a = \frac{4}{1.15^4}$, and the ratio between the terms, $x = 1/1.15$. Use equation (1.5) to find the sum.

$$S_\infty = \frac{a}{1 - x} \tag{1.5}$$

It gives,

$$P_0 = \frac{2}{1.15} + \frac{2.5}{1.15^2} + \frac{3}{1.15^3} + \frac{4/1.15^4}{1 - 1/1.15} = \$23.14$$ ♥

After the payment of the first $2 dividend, the next dividend will be $2.50 available one year later, $3.00 two years later and so on. To visualize the cash flows, draw another table as follows.

Year	1	2	3	4	5	6	...	∞
Dividend	$2.50	$3.00	$4.00	$4.00	$4.00	$4.00	...	$4.00

Proceeding on the same lines as before, we get the present value of the cash flows

$$P_1 = \frac{2.5}{1.15} + \frac{3}{1.15^2} + \frac{4}{1.15^3} + \frac{4}{1.15^4} + \frac{4}{1.15^5} + \frac{4}{1.15^6} + \ldots \infty$$

Starting with the third term, it becomes an infinite geometric series, with $a = 4/1.15^3$, and $x = 1/1.15$. Use again to get

$$P_1 = \frac{2.5}{1.15} + \frac{3}{1.15^2} + \frac{4/1.15^3}{1 - 1/1.15} + \ldots \infty = \$24.61$$ ♥

Why does the value of the stock increase after a year? This is because the investors are expecting to receive a higher set of dividends. This becomes clear when we compare the two timelines and the cash flows.

Problems

3.13. Philadelphia Electric Co. (now Exelon) bonds were once selling at 120.25, with 27 years to maturity and semiannual interest payments. The coupon rate was 18%. If your required rate of return were 16% at the time, would you have bought these bonds?
B = $1,123.04, no ♥

3.14. The Somerset Company bonds have a coupon of 9%, paying interest semiannually. They will mature in 10 years. However, because of poor financial health of the company you do not expect to receive more than 5 interest payments. Also, you do not expect to receive more than 50% of the principal, after 4 years. Your required rate of return is 12%. What is the maximum price you are willing to pay for these bonds? $503.26 ♥

3.15. Gambia Express bonds have a coupon of 8%, pay interest semiannually, have a face value of $1,000 and will mature after 10 years. Your income tax rate for interest income is 40%, but only 16% on capital gains. You pay the taxes once a year. How much should you pay for a Gambia bond if your after-tax required rate of return is 10%?
$666.85 ♥

3.16. Leo Corporation bonds have a coupon of 9%; they pay interest semiannually; and they will mature in 6 years. You pay 30% tax on ordinary income and 20% on capital gains. What price should you pay for a Leo bond so that it gives you an after-tax return of 15%? $647.78 ♥

3.17. In 2008, Rumsfeld Co 13s2027 bonds paid interest annually, and their price was quoted as 98. You had to pay 28% tax on interest income and capital gains, and your required after-tax rate of return was 10%. Do you think you would have bought these bonds as a long term investment? No, B = $943.90 ♥

3.18. Caruso Corporation 9% bonds will mature on January 15, 2019. They pay interest semiannually. On July 16, 2007, these bonds are quoted as 87.375. If your required rate of return is 11.5%, should you buy these bonds? No, B = $842.70, they sell at $873.75 ♥

3.19. Cincinnati Corporation 9% bonds pay interest semiannually, on April 15 and October 15, and they will mature on April 15, 2019. They are selling at 89 on October 16, 2008. Considering its risk characteristics, your required rate of return for this bond is 10%.

(A) Do you think you should buy this bond? Yes, B = \$935.89 ♥
(B) Suppose you buy the bond at the market price, what is its approximate yield to maturity? 10.63% ♥
(C) Use Excel, Maple, or WolframAlpha to find its exact yield to maturity. 10.77% ♥

3.20. HAL Inc. stock is selling for \$121 per share and it just paid an annual dividend of \$4.40. According to your careful analysis, you feel that HAL will continue to grow at the rate of 25% per year for the next three years and then it will maintain a growth rate of 10% per year forever. Its dividend payout ratio is expected to remain constant. Would you invest your money in HAL, if your required rate of return is 16%?
S = \$116.29, don't buy. ♥

3.21. White Rock Company stock just paid an annual dividend of \$2.00. The dividends are expected to grow at the rate of 3% annually for the next 10 years. After 10 years White Rock will stop growing altogether, but will continue to pay dividends at a constant rate. What is the price of this stock, assuming 12% discount rate. \$20.20 ♥

3.22. Baffin Corporation stock has just paid the annual dividend of \$4.00. The company is expected to grow at the rate of 10% (along with its dividends) for the next three years, then it is expected to grow at the rate of 3% forever. The investors require a return of 12% for their investment in the Baffin stock. What is the fair market price of a share of the stock? \$54.95 ♥

3.23. Timon Corporation stock has just paid the annual dividend of \$2. This dividend is expected to grow at the rate of 5% per year for the next ten years, and then it will remain constant. If your required rate of return is 12%, how much should you pay for a share of Timon stock? \$23.01 ♥

3.24. McCormack Corporation just paid the annual dividend of \$4.00. The dividends are expected to have a growth rate as follows: g_1 = 7%, g_2 = 5%, g_3 = 3%, $g_4 = g_5 = ... = 0$, where the g's represent the growth in the first year, second year, etc. Your required rate of return is 10%. How much should you pay for a share of McCormack stock? \$45.86 ♥

3.25. Carpenter Corporation is expected to pay \$2.00 dividend after one year, \$3.00 after 2 years, \$4.00 after 3 years, and then \$5.00 a year uniformly after fourth and subsequent years. If the stockholders of Carpenter require 12% return on their investment, find the price of the stock now. What is its price just after the payment of the first \$2.00 dividend?
\$36.68, \$39.08 ♥

3.26. Clifford Corporation stock is expected to pay a dividend on every January 25. In 2008, the dividend is \$3.00, in 2009 \$3.25, in 2010 \$3.50, and in 2011 and all the subsequent years it is expected to be $4.00. The shareholders of Clifford require a return of 13% on their investment. Find the price of this stock on January 14, 2008, just before it pays its dividend. What is its price on January 28, 2010, just after it has paid its dividend?

$32.71, $30.77 ♥

Multiple Choice Questions

1. For a bond selling at its face value, 5 years before maturity,

A. the yield to maturity equals its current yield
B. the bond should have zero coupon
C. the coupon rate is more than its current yield
D. the coupon rate is equal to the prime rate

2. For the yield-to-maturity of a bond to be equal to its current yield,

A. the bond must be selling at a discount
B. the bond should have zero coupon
C. the bond must sell at its face value
D. coupon rate must be equal to the prime rate

3. A bond is listed in WSJ as Ford 8.5s17 and priced as 85. This means

A. Its semiannual interest payment is $85
B. Its maturity date is unknown
C. Its price is $85
D. Its current yield is 10%

4. For a perpetual bond,

A. It is not possible to calculate its current yield
B. The face amount is unknown
C. The market price is inversely proportional to the interest rates
D. The coupon rate is not known

5. Gordon's growth model does not assume that

A. the rate of growth of dividends is constant
B. the required rate of return by the stockholders is greater than the rate of growth of the company
C. the company must pay all its earnings in dividends
D. the growth will continue forever

6. For a zero-coupon bond,

A. The coupon rate is not known
B. The face amount is not known
C. The market price is directly proportional to the interest rates
B. It is possible to calculate its current yield

7. For a bond that is selling at par

A. The face value is not equal to its market value
B. The current yield is less than its yield to maturity
C. The coupon rate is equal to its current yield
D. The coupon rate is more than its yield to maturity

8. For a zero-coupon bond

A. The market price is more than its face value
B. The current yield is zero
C. The yield to maturity is zero
D. The risk of the bond is zero

9. The prime rate is the rate used by

A. Stock brokers for loans to their customers
B. Federal Reserve for loans to member banks
C. Banks as a benchmark rate for commercial lending
D. Treasury Department for short-term borrowing

Key Terms