Real Estate Investments with Stochastic Cash Flows

Riaz Hussain

Kania School of Management
University of Scranton
Scranton, PA 18510
hussain@scranton.edu
570-941-7497

April 2006

JEL classification: G12
Key words: Real estate, Uncertainty, Stochastic methods, Asset valuation

Real Estate Investments with Stochastic Cash Flows

Abstract

This paper examines the ownership of real estate as a long-term, risky investment. Using stochastic calculus, the risk is analyzed by assuming that the cash flows in a property investment are growing as arithmetic Brownian motion with the possibility of becoming negative, while the value of the property is growing as a geometric Brownian motion. The analysis takes into account depreciation and taxes. The results are useful for a corporation or a long-term individual investor interested in real-estate investments.

1. Introduction

Both individuals and corporations invest in real estate. A family may invest in a home to live. A landlord will invest in a rental property to earn a living. A corporation can invest in a shopping mall on behalf of its stockholders. A university has to invest in a parking garage to alleviate the parking problem on campus.

A real estate investment is usually considered to be a safe investment. A bank may lend up to eighty percent of the value of a house to a homeowner, but the stockbrokers can lend only up to fifty percent of the stock purchased on margin. The stockbroker can monitor the price of a stock every day and send a margin call as soon as the value of the stock drops below a certain level.

A real estate holding should be a long-term investment for a firm or an individual. Brown and Geurts (1) investigate the holding period of real estate properties by individual investors. By analyzing the real estate transactions in San Diego, they find that the average holding period by the investors is somewhat less than five years. The investor behavior is contrary to the theoretical calculations in this paper, which demonstrates that the optimal holding period is much longer.

In another paper, Brown (2) explores the reasons for owning real estate by private investors. In particular he examines the risk peculiar to real estate investments, including the entrepreneurial abilities of the owner. Geltner and Miller (3) look at the risk in the real estate investments for individuals and try to measure it in terms of CAPM. Their conclusion is that one cannot really do so.

Some researchers focus on the institutional investors and their investments in real estate. Chun, et.al. (4) report that the institutional investors hold a surprisingly small fraction, perhaps 2 or 3%, of their investments in real estate assets. In their estimation, if CAPM is the proper model to assess risk, this fraction should be about 12%.

French and Gabrielli (5) look at the overall concept of risk as applied to real estate investments. They consider the uncertainty in the valuation of British real estate. One simple way to assess the risk of real estate investments is to look at the beta of REITs. According to Corgel and Djoganopoulos (6), who examine the β of 60 REITs, the mean β is about 0.36, implying the relative safety of such investments. They also stress the special characteristics of REITS that contribute to their low betas.

It is also possible to treat the uncertainty in the real estate valuation in terms of stochastic variables. For example, Buttimer and Ott (7) assume that the spot lease price follows a geometric mean-reversion stochastic process.

Consider a corporation that buys an apartment building as an investment. The two principal sources of financial risk are (1) the uncertain cash flows during the holding period, and (2) uncertain final selling price of the property. This paper considers both these risks in terms of stochastic variables and obtains a closed-form solution to the problem.

2. Investment under Certainty

Consider a firm that has invested in a rental property, where the cash flows are known with certainty. The firm wants to hold the asset for a certain time τ, and then sell it. It expects that the property will produce some rental income, and that the property will be sold at a profit later.

Consider time to be a continuous variable. Suppose we can collect the rents continuously, and likewise pay all the bills continuously. Let us define x per period as follows:

$$x = \text{rental income} - \text{operating expenses} - \text{R. E. taxes} \tag{1}$$

The rental income includes the rents collected on a monthly basis; however, we look at them as a continuous income stream. Similarly, we assume that all the operating expenses are paid continuously. These could be maintenance expenses or utility bills. Similarly the real estate taxes are paid continuously. It is possible that x can have a negative value, particularly if the property remains vacant. We are not including interest payments, or income tax payments in x. We make the assumption that x is growing linearly with time at the rate a.

A. Cash flows are growing linearly with time

The total transaction cost at the time of the purchase of the property consists of fixed cost b (lawyer's fee, deed recording fee, document preparation fee, etc.) and the variable cost βH, where β is the rate of transfer taxes and H is the purchase price. Let us assume that the firm is able to get tax benefit of these costs immediately. Thus the after-tax transaction cost is

$$(b + \beta H)(1 - \xi) \tag{2}$$

where ξ is the income tax rate of the firm. We further assume that the value of the structure is ϕH and that of the land $(1 - \phi)H$. For tax purposes, assume that the depreciable life of the property is N years. Using straight-line method, the depreciation per year is $\phi H/N$. The after-tax cash flow for this investment is thus

$$C(t) = (x + at)(1 - \xi) + \phi\xi H/N, \quad t < N \tag{3}$$

$$C(t) = (x + at)(1 - \xi), \qquad t > N \tag{4}$$

There is a discontinuity in the cash flows at the time the property is completely depreciated. Suppose the cash flow x = \$9000 annually, as defined by (1). It grows at an annual rate of 3%, meaning a = .03*9000 = \$270 per year. Suppose the purchase price of the house, H = \$100,000. This includes the value of the land, assumed to be \$20,000. The value of the structure is \$80,000 and thus $\phi = .8$. Assume the income tax rate is 30%, or $\xi = .3$. The depreciable life of the house is assumed to be 27.5 years, that is, N = 27.5 years. The following diagram represents the cash flows in this case.

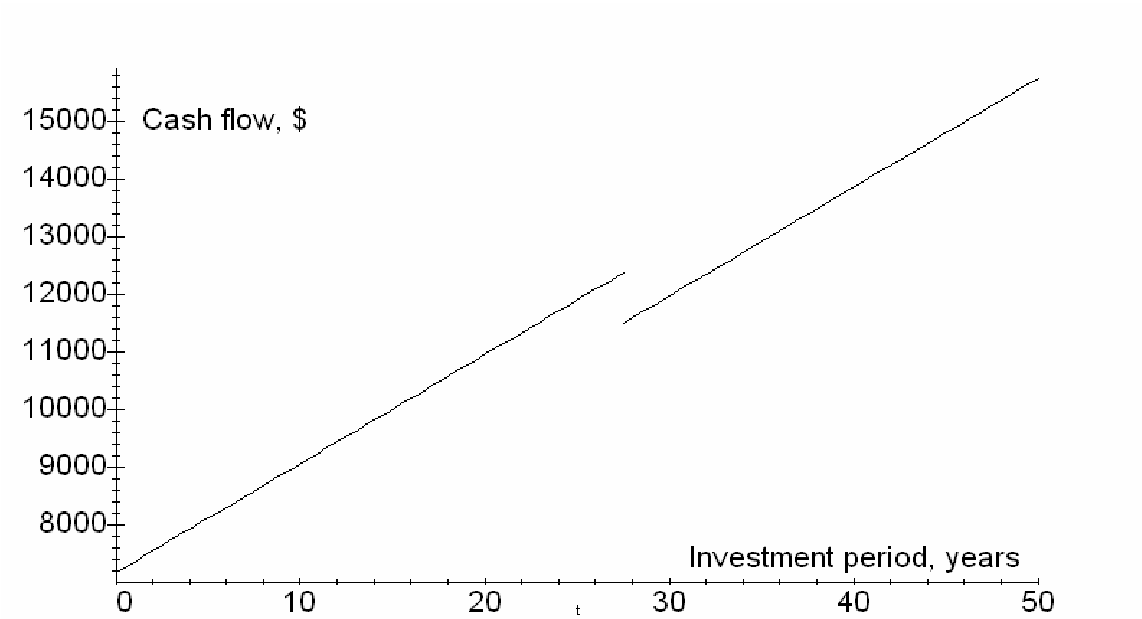

Figure 1: Assumed to be growing linearly, the cash flows from a real estate investment show a discontinuity at a point when the depreciation of the asset is no longer available. In this case the depreciable life of the asset is 27.5 years.

Consider $(x + at)$ as uniformly growing cash flow. Later, we will take x to be a stochastic variable. Let us assume that the after-tax discount rate is r, which is possibly equal to the weighted average cost of capital for the firm. In the present analysis, we assume that the firm will hold the property for τ years, which is less than the depreciation period of the property. That is, $\tau < N$. In an extension of this paper we shall include an arbitrary holding period that could be more than the depreciable life of the property.

Assume that the property value is growing exponentially at the constant rate R. Later, we will consider the case when R is a stochastic variable. The final value of the property, after time τ, is thus $He^{R\tau}$. Again, we can write the transaction costs as $c + \gamma He^{R\tau}$, where c is the fixed part and γ represents the variable component. The quantity γ is the sum of realtor's selling commission and the transfer taxes. For instance, if the transfer taxes are 1.85% and the realtor's fee is 6% then γ equals .0785.

The book value of the property after τ years is $H - \tau\phi H/N$. The taxes payable on the capital gains are thus $[He^{R\tau} - (c + \gamma He^{R\tau}) - (H - \tau\phi H/N)]\xi$.

The net cash flow after taxes from the sale of the property is found as follows:

$$= He^{R\tau} - (c + \gamma He^{R\tau}) - [He^{R\tau} - (c + \gamma He^{R\tau}) - (H - \tau\phi H/N)]\xi$$

$$= [(1-\gamma)(1-\xi)e^{R\tau} + (1 - \tau\phi/N)\xi]H - c(1-\xi) \qquad (5)$$

We can view the investment in the property as having two components, namely, the continuous cash flows from the net rental income and the net capital gains. Combining them, we can find the net present value of the investment as

$$\text{NPV} = -H - (b + \beta H)(1-\xi) + \int_0^\tau [(x + at)(1-\xi) + \phi H\xi/N]e^{-rt}\,dt$$
$$+ \{[(1-\gamma)(1-\xi)e^{R\tau} + (1 - \tau\phi/N)\xi]H - c(1-\xi)\}e^{-r\tau} \qquad (6)$$

This gives

$$\text{NPV} = \left(\left[\frac{xr + a}{r^2}\right](1-\xi) + \frac{\phi\xi H}{rN}\right)(1 - e^{-r\tau}) - \frac{(1-\xi)a\tau e^{-r\tau}}{r}$$
$$- H - (b + \beta H)(1-\xi) + \{[(1-\gamma)(1-\xi)e^{R\tau} + (1 - \tau\phi/N)\xi]H - c(1-\xi)\}e^{-r\tau} \qquad (7)$$

Setting the right hand side of the above equation equal to zero and solving for H, we get the critical value of the property, where the NPV is zero, to be

$$H^* = \frac{N(1-\xi)[a + xr - br^2 - (a + xr + cr^2 + ar\tau)e^{-r\tau}]}{r\{[(\phi - rN + \phi r\tau)\xi - (1-\xi)(1-\gamma)rNe^{R\tau}]e^{-r\tau} + rN(1+\beta) - (\phi + rN\beta)\xi\}} \qquad (8)$$

The investment in the property will be a profitable one if the firm can buy, or build, the property for less than H^* as given by (8)

Suppose the income tax rate is zero. This may be the case if a university decides to build a dormitory for the students. Then for $\xi = 0$, (8) becomes

$$H^* = \frac{a + xr - br^2 - (a + xr + cr^2 + ar\tau)e^{-r\tau}}{r^2[1 + \beta - (1-\gamma)e^{(R-r)\tau}]} \qquad (9)$$

If there are no transaction costs, then $b = \beta = c = \gamma = 0$. Then (9) becomes

$$H^* = \frac{a + xr - (a + xr + ar\tau)e^{-r\tau}}{r^2[1 - e^{(R-r)\tau}]} \tag{10}$$

To appreciate the utility of (8), let us consider a numerical example. A firm buys a house, which creates a positive cash flow of $800 a month. This amount is after paying operating expenses and real estate taxes, but before paying interest and income taxes. The net rent will increase by $24 per month every year. Assume that the cash flows occur uniformly throughout the year. The land value is $20,000. The total value of the house will rise exponentially by 5% annually. The total depreciation period is 27.5 years, and the applicable tax rate is 28%. The proper discount rate is 10%. A firm wants to buy the house, rent it for 12 years, and then sell it. We need to calculate the purchase price of the house, which will make its NPV = 0.

If the total value of the house is H, then $\phi = (H - 20{,}000)/H$, $x = 800*12 = 9600$, $\alpha = .03*800*12 = 288$, $r = .1$, $R = .05$, $\tau = 12$, $\xi = .28$, $b = 200$, $c = 200$, $\beta = .0185$, $\gamma = .0785$, and $N = 27.5$. Substituting these numbers in (8), we find the NPV = 0 when the purchase price of the house is $102,586.

By setting the derivative of NPV with respect to τ equal to zero, we can find an optimal time to hold the investment. For a given set of data, the optimal holding period may be between 0 and N. Using the previous numerical values, we find the optimal holding period for the property to be 5.322 years. In particular, we note that the linear growth in the rents is 1% of the initial rent.

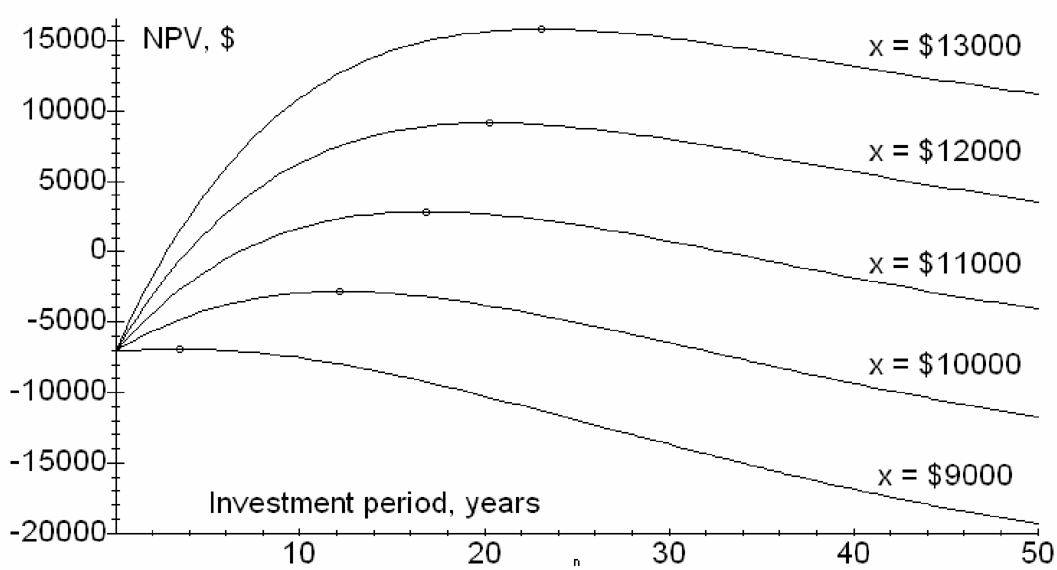

Figure 1: The NPV for different investment periods. The variables in this diagram are H = 100,000, $\phi = .8$, $r = .1$, $N = 27.5$, $\xi = .3$, $R = .05$, $a = .01x$, $b = 200$, $c = 200$, $\beta = .0185$, $\gamma = .0785$. The optimal holding period and the maximum NPV are indicated by small circles on the curves. The corresponding numerical values are given in Table 1.

Next, we consider a higher rate of growth in the rents, namely 3%. As a result, the curves in the next diagram are higher and they reach a peak later. The results are available in Figure 2, and Table 1.

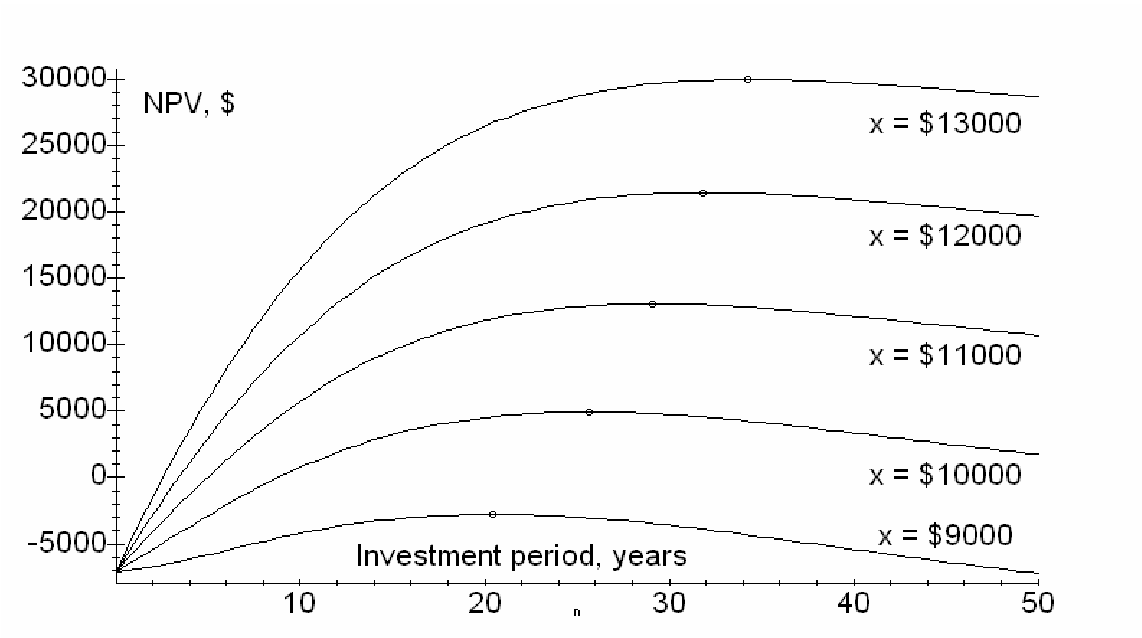

Figure 2: The NPV for different investment periods. The variables in this diagram are H = 100,000, ϕ = .8, r = .1, N = 27.5, ξ = .3, R = .05, a = .03x, b = 200, c = 200, β = .0185, γ = .0785. The optimal holding period in this case is 27.5 years, the depreciable life of the property.

We summarize the numerical results in a table as follows. We assume that the cash flows, defined by (1) have annual values from \$9000 to \$13,000 and that they are growing linearly at the rate of 1% and 3% annually. The investment is never profitable for x = \$9000. The optimal holding periods get longer for higher cash flows and for larger growth rates.

x	a = .01x		a = .03x	
	Optimal holding period, years	Maximum NPV, $	Optimal holding period, years	Maximum NPV, $
$9,000	3.449	-6908	20.43	-2777
$10,000	12.18	-2817	25.68	4943
$11,000	16.81	2849	29.14	13,094
$12,000	20.27	9143	31.82	21,459
$13,000	23.09	15,789	34.23	29,969

Table 1: The values of the parameters are the same as those in Figure 1 and 2. With a higher growth rate in rents, the optimal period gets longer, and the maximum NPV also rises.

3. Valuation with Stochastic Variables

A. Valuation of cash flows

In the previous section we assumed that the cash flows are certain. Others have used stochastic variables to describe the valuation of real estate. Buttimer and Ott (7) assume that the spot lease price follows a geometric mean-reversion stochastic process. This type of process reflects a market where supply and demand adjust to a long-run equilibrium over time. The evolution of the spot lease price $P(t)$ is described by

$$\frac{dP(t)}{P(t)} = \{\alpha_P + [\eta_P (\bar{P}e^{\alpha_P t} - P(t))]\}\ dt \tag{11}$$

They find numerical solution for the resulting differential equations to get meaningful results.

Consider the two components of the real estate investment: the cash flows and the capital gains. Assume that the cash flows follow an arithmetic Brownian motion, while the capital gains assume the growth as a geometric Brownian motion. We separate the two components and evaluate them individually.

Further, assume that the cash flows, defined by (1), are not known with certainty, rather they follow arithmetic Brownian motion. The stochastic differential equation for this process is

$$dx = \alpha dt + \sigma dz \tag{12}$$

In this equation, α is the rate of growth for the cash flows, σ is the standard deviation of the cash flows, and z is a standard Wiener process. In general, x can have negative or positive values. From stochastic calculus, we get

$$dx^2 = \sigma^2\ dt \tag{13}$$

Suppose $V(x,t)$ is the value of a security, which has the same cash flows as the cash flows of the real estate investment. We may write

$$dV(x,t) = \frac{\partial V}{\partial x}dx + \frac{1}{2}\frac{\partial^2 V}{\partial x^2}dx^2 + \frac{\partial V}{\partial t}dt \tag{14}$$

Combining (12), (13), and (14), we have

$$dV(x,t) = \frac{\partial V}{\partial x}(\alpha dt + \sigma dz) + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial x^2}dt + \frac{\partial V}{\partial t}dt \tag{15}$$

Expected value of cash flows,

$$\mathrm{E[CF]} = \alpha \frac{\partial V}{\partial x} dt + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial x^2} dt + \frac{\partial V}{\partial t} dt \tag{16}$$

Using (3) and (4), we can find the expected after-tax cash flow as

$$Cdt = x(1 - \xi)\, dt + \phi\xi H/N\, dt,\ 0 < t < N \tag{17}$$

We ignore the expected value of capital gains, E[CG], for the time being, but we will include their contribution later. Combining (16) and (17), we get the total return in a time interval dt and equate it to the total return of the security V as

$$rVdt = \alpha \frac{\partial V}{\partial x} dt + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial x^2} dt + \frac{\partial V}{\partial t} dt + x(1 - \xi)\, dt + \phi\xi H/N\, dt,\ 0 < t < N \tag{18}$$

Assume that the firm will keep the property for τ years and then sell it. Let us introduce the variable τ as the time to maturity of these cash flows, that is $t = -\tau$. When $\tau = 0$, the cash flows will cease. With the change in variable, we can write (18) as

$$\frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial x^2} + \alpha \frac{\partial V}{\partial x} - rV + x(1 - \xi) + \phi\xi H/N = \frac{\partial V}{\partial \tau},\ 0 < \tau < N \tag{19}$$

The above equation (19) represents a boundary value problem for $V(x,\tau)$. Let us assume that the firm has the option to abandon the asset when the cash flows become sufficiently negative. Suppose the critical point is at $x = q$. Thus the two boundary conditions are

$$V(x,\tau) = 0 \text{ when } \tau = 0 \tag{20}$$

$$V(x,\tau) = 0 \text{ when } x = q \tag{21}$$

We can solve (19) under the boundary conditions (20, 21). The result, in terms of error functions and exponential functions, is

$$V(x,\tau) = \frac{A_1(1 + F_1)E_1 + A_2(1 + F_2)E_2 + A_3(1 + F_3)E_3 + A_4(1 + F_4)E_4 + A_5}{2Nr^2} \tag{22}$$

Where

$$A_1 = [\alpha(1 + r\tau) + r(2q - x)](1 - \xi)N + \phi\xi rH \tag{23}$$

$$A_2 = -[\alpha(1 + r\tau) + r\,x)](1 - \xi)N - \phi\xi rH \tag{24}$$

$$A_3 = A_4 = -(\alpha + rq)(1 - \xi)N - \phi\xi rH \tag{25}$$

$$A_5 = 2(\alpha + rx)(1 - \xi)N + 2\phi\xi rH \tag{26}$$

$$F_1 = \mathrm{erf}\left(\frac{\alpha\tau + q - x}{\sigma\sqrt{2\tau}}\right) \tag{27}$$

$$F_2 = \mathrm{erf}\left(\frac{\alpha\tau - q + x}{\sigma\sqrt{2\tau}}\right) \tag{28}$$

$$F_3 = \mathrm{erf}\left(\frac{q - x - \tau\sqrt{\alpha^2 + 2r\sigma^2}}{\sigma\sqrt{2\tau}}\right) \tag{29}$$

$$F_4 = \mathrm{erf}\left(\frac{q - x + \tau\sqrt{\alpha^2 + 2r\sigma^2}}{\sigma\sqrt{2\tau}}\right) \tag{30}$$

$$E_1 = \exp\left(\frac{2\alpha(q - x)}{\sigma^2} - r\tau\right) \tag{31}$$

$$E_2 = \exp(-r\tau) \tag{32}$$

$$E_3 = \exp\left(\frac{(q - x)(\alpha - \sqrt{\alpha^2 + 2r\sigma^2})}{\sigma^2}\right) \tag{33}$$

$$E_4 = \exp\left(\frac{(q - x)(\alpha + \sqrt{\alpha^2 + 2r\sigma^2})}{\sigma^2}\right) \tag{34}$$

The error function is defined as

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\,dt \tag{35}$$

The solution (22) equals zero when $x = q$. It satisfies the PDE (19). Further, when $\alpha = a$ and $\sigma = 0$, (22) reduces to the cash-flow part of (7). That is,

$$V(x,\tau)\Big|_{\sigma = 0} = \left(\left[\frac{xr + a}{r^2}\right](1 - \xi) + \frac{\phi\xi H}{rN}\right)(1 - e^{-r\tau}) - \frac{(1 - \xi)a\tau e^{-r\tau}}{r} \tag{7a}$$

B. Valuation of Price Appreciation

Let us assume that the value of the property is growing at the continuously compounded rate R with standard deviation ω. We also assume that the returns have a lognormal distribution. The expected value H_τ of the asset after time τ will be

$$H_\tau = H\exp\{(R + \omega^2/2)\tau\} \tag{36}$$

After paying the selling costs and income taxes at rate ξ, and considering the tax benefits of depreciation, the final value of the property, F, becomes

$$F = H_\tau - [H_\tau - (c + \gamma H_\tau) - (H - \tau\phi H/N)]\xi \tag{37}$$

The present value of capital gain, G, is thus

$$G = -H - (b + \beta H)(1 - \xi) + \{H_\tau - [H_\tau - (c + \gamma H_\tau) - (H - \tau\phi H/N)]\xi\}e^{-r\tau} \tag{38}$$

Using (16), the NPV in this case is

$$\text{NPV} = V(x,\tau) + G \tag{39}$$

In (39), G is given by (38) and $V(x,\tau)$ by (22). If there is no uncertainty in the cash flow, and the final value of the property is known with certainty, then $\sigma = \omega = 0$. Substituting these values in (39), we get previous result (7).

Consider a numerical example again. Suppose the total value of the property is H, then $\phi = (H - 20{,}000)/H$. Let us assume further that $x = 800*12 = 9600$, $\alpha = .03*800*12 = 288$, $r = .1$, $R = .05$, $\tau = 12$, $\xi = .28$, $b = 200$, $c = 200$, $\beta = .0185$, $\gamma = .0785$, and $N = 27.5$. Substituting these numbers in (39), we find the NPV = 0 when the purchase price of the property is \$112,002. We should compare it with the previous value \$102,586, obtained from (8).

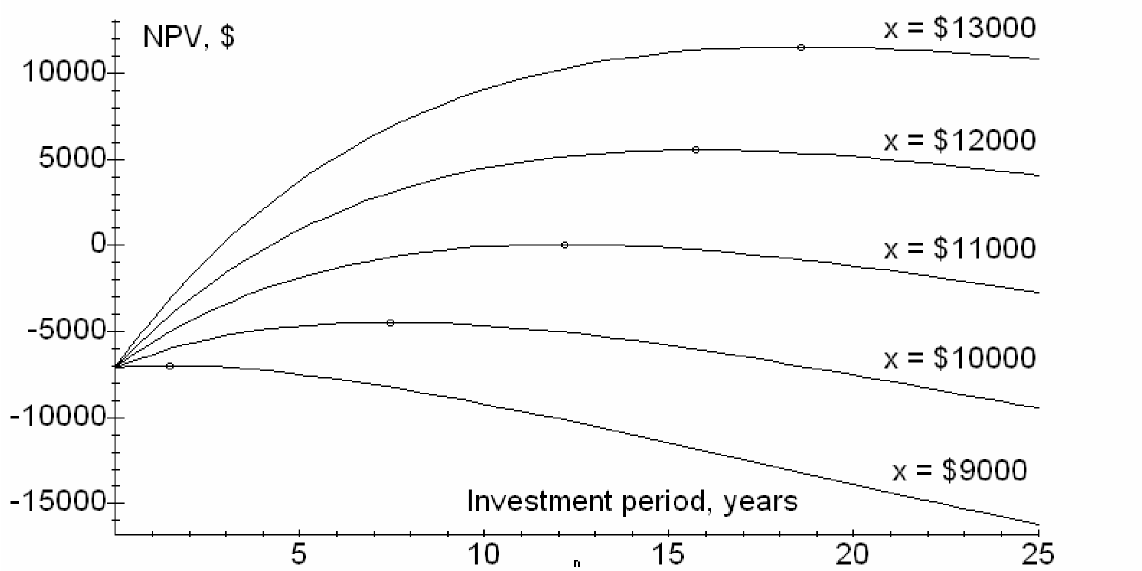

Figure 3: The NPV of a property with following parameters: $H = 100{,}000$, $\phi = .8$, $r = .1$, $N = 27.5$, $\xi = .3$, $\omega = R/2$, $R = .05$, $\alpha = .01x$, $\sigma = x/2$, $b = 200$, $c = 200$, $\beta = .0185$, $\gamma = .0785$, and $q = 0$. The optimal holding periods are indicated by small circles at the peak of curves.

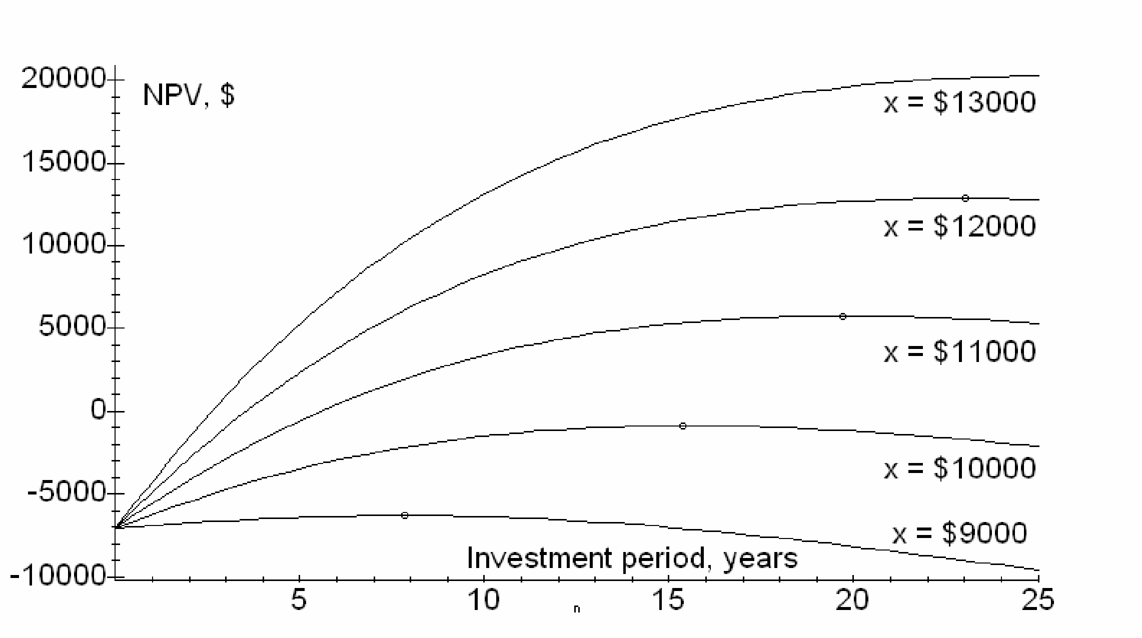

Figure 4: The NPV of a property with following parameters: H = 100,000, ϕ = .8, r = .1, N = 27.5, ξ = .3, $\omega = R/2$, R = .05, $\alpha = .03x$, $\sigma = x/2$, b = 200, c = 200, β = .0185, γ = .0785, and q = 0.

The optimal holding period satisfies the condition

$$\frac{\partial \mathrm{NPV}}{\partial \tau} = 0 \tag{40}$$

Solving the equation with suitable numerical values of various parameters, we get the optimal holding period where the NPV is maximized. These values are depicted as small circles at the top of the curves in Figures 1-4.

The following table summarizes the numerical results of this section.

x	$\alpha = .01x$		$\alpha = .03x$	
	Optimal holding period, years	Maximum NPV	Optimal holding period, years	Maximum NPV
$9,000	1.454	-$6970	7.817	-$6308
$10,000	7.461	-$4469	15.37	-$875
$11,000	12.16	$61	19.72	$5733
$12,000	15.71	$5539	23.03	$12,853
$13,000	18.57	$11,536	27.5	$20,227

Table 2: The table provides the optimal holding period and the maximum NPV for real estate investments. It assumes the drift parameter α to be 1%, and 3% of the initial cash flow.

C. Abandonment Option

As a boundary condition, we have assumed that the owner should abandon an asset if the cash flows become sufficiently negative. This could happen if the vacancy rate, operating expenses, and real estate taxes are too high. The optimal value of q to abandon the asset is when

$$\frac{\partial \mathrm{NPV}}{\partial q} = 0 \tag{41}$$

We can carry out the analysis by performing the differentiation, and solving the above equation for q. Table 3 shows the result of this calculation under different variables. For instance, if we start with an initial cash flow of \$12,000 annually, it is better to abandon the asset if the cash flows become –\$14,343 annually. The probability of reaching that number, through random fluctuations of cash flows, is quite low. From a practical point of view we assume $q = 0$ while plotting the curves in Figure 3 and 4. The impact of q on the NPV in (39) is also very small.

Cash flow x	Optimal q to quit
\$9000	–\$11,051
\$10,000	–\$12,149
\$11,000	–\$13,246
\$12,000	–\$14,343
\$13,000	–\$15,441

Table 3: It presents the optimal value of the cash flow q when the owner of real estate should abandon the asset.

4. Conclusions

This article presents a framework for the valuation of a real estate property with the assumption that the risk of a real estate investment can be captured through the use of stochastic variables for the cash flows and the final value of the property. Using reasonable assumptions, the paper develops a closed-form solution (39) of the NPV of an investment, including the abandonment option. It concludes that the investments are indeed long-term commitments. Most individuals have short-term investment horizons and thus they are unable to realize the full potential of these investments.

The paper should include investment periods longer than the depreciable life of the asset. It should show some empirical evidence for the basic assumptions and the numerical values of the parameters employed in actual calculations. However, a forthcoming paper will present these extensions.

5. Glossary of symbols

a = linear rate of increase of cash flow x with time
b = fixed cost at the time of purchase of property
c = fixed cost when the property is sold
G = present value of the capital gains on the property
H = purchase price of the property
N = depreciable life of the property
q = the value of the cash flow when the firm should abandon the property
r = risk adjusted discount rate for this investment
R = compound rate of appreciation of the value of the property per period
$V(x,\tau)$ = present value of the cash flows x for a period τ
x = rental income – operating expenses – R. E. taxes
α = the drift term in the stochastic cash flows
β = transfer tax rate
β = transfer tax rate
β = transfer tax rate, or variable cost at the time of purchase of property
ϕ = fractional value of the structure of the property
= value of structure only/(value of structure + land)
γ = total of transfer taxes and realtor's selling commission rate
σ = standard deviation of the cash flow per period
τ = period of investment in the property, $\tau < N$
ξ = income tax rate of the firm

6. References

1. Brown, Roger J. and Tom G. Geurts, "Private Investor Holding Period", Journal of Real Estate Portfolio Management; May-Aug 2005; 11, 2; pg. 93

2. Brown, Roger J., "Risk and Private Real Estate Investments", Journal of Real Estate Portfolio Management; May-Aug 2004; 10, 2; pg. 113

3. Geltner, D. M. and N. G. Miller, *Commercial Real Estate Analysis and Investments*, Prentice Hall, Upper Saddle River, NJ, 2001

4. Chun, Gregory H., J. Sa-Aadu, and James D. Shilling, "The Role of Real Estate in an Institutional Investor's Portfolio Revisited," Journal of Real Estate Finance and Economics, 29:3, 295-320, 2004

5. French Nick and Laura Gabrielli, "The Uncertainty of Valuation", Journal of Property Investment & Finance; 2004; 22, 6; pg. 484

6. John B Corgel, Chris Djoganopoulos, "Beta of REITS", Financial Analysts Journal, Charlottesville: Jan/Feb 2000, Vol. 56, Iss. 1; pg. 70

7. Buttimer, Richard and Steven H. Ott, “Commercial Real Estate Valuation and Development”, working paper, December, 2004