Risk Averse Speculation and Exchange Rate Determination: An Econometric Analysis of the U.S. Dollar versus Euro, Pound, and Yen

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October 2005

Presented at the 32nd Annual Conference of the Northeast Business & Economics Association at Hyatt Regency Newport on Goat Island, Newport, R.I., U.S.A., October 28-29, 2005
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Abstract*1

The objective of this analysis is to determine the exchange rate between the U.S. dollar and three other major currencies, the euro, the pound, and the yen by looking at the real rate of return and risk that financial assets have in these four economies. Risk averse speculators will try to maximize their return and minimize their risk by investing in different countries, but these capital flows will affect the value of these four currencies (their exchange rates). The empirical results show that before 2001 the real return in the U.S. was high and the dollar was appreciated; after 2001, the same real return became negative and the dollar was depreciated, but after 2004 the real returns have growing positively, so the dollar is expected to appreciate. Then, by forecasting risk and return in countries’ assets, we can determine the value of these currencies (exchange rates) in the future.

1. Introduction

It is well established that the volatility of exchange rates displays considerable persistence. That is, large movements in spot rates tend to be followed by more large movements later, which increasing risk and producing serial correlation in real returns. Thus, past volatility and current one can be used to predict future volatility and the forward discount or premium of the different currencies. Investors in foreign assets must pay attention not only to the expected return from their investment activity, but also to the risk that they incur. Risk averse investors try to reduce their exposure during periods of high volatility by predicting the real return of their investment and the volatility (variance) of this return. This volatility can be forecasted with a GARCH (p, q) model or a genetic program, which give broadly similar results. Investors will invest in assets denominated in a currency that its real return will be higher than the others and its risk to be the smallest one. Determining these assets with the highest real return and lowest risk, we can determine the exchange rate of this specific country. An excess demand for the country’s assets will appreciate its currency.

Some recent facts (“news”) reveal the effect of speculation on the different exchange rates. On Tuesday February 22, 2005, South Korea’s Central Bank announced that plans to diversify its foreign exchange reserves, which traders took to mean a slowdown in purchases of dollar-denominated securities. The U.S. dollar fell to $1.3259 per euro and lost value with respect the other major currencies. The DJIA slid 174.02 points (1.6%) as concerns about the weak dollar sparked a sell-off of the U.S. currency. Also, Gold surged $7.40 to $434.50 and oil

* We would like to acknowledge the help provided by our research assistants, Matthew Horejs and Robert Kalaf. Financial support from Henry George Research Funds is gratefully acknowledged. The usual disclaimer applies.

*1 Their standard deviations are: \( \sigma_{\text{euro}} = 0.145 \), \( \sigma_{\text{UKpound}} = 0.335 \), and \( \sigma_{\text{yen}} = 74.711 \).

3 See, Kallianiotis (2004a and b).

4 See, Neely and Weller (2002).
climbed to $51.42 per barrel. In addition, terrorist attacks globally rose last year (2004) to about 650 from 175 in 2003, said congressional aides briefed by State Department and intelligence officials.

A terrorist attack in London on July 7, 2005 caused stocks worldwide to fall; the London stocks (FTSE 100 index) fell by 200 points, the DJIA fell by 250 points, U.K. pound slumped to $1.7403 from $1.7556, bonds gained (10-year AAA=4.80%), oil in N.Y. fell by $5 to $57, and gold price increased by $4 to $430 per troy ounce; but after this shudder in the markets, they rebounded quickly.

On Monday, October 3, 2005, Turkey “invaded” EU and we were expected to see some effects on euro, but nothing happened; it did not change at all. Then, invasions have no effects on exchange rates, only speculations do. Stranger world and it is becoming worse every day! Economists and all social scientists will have a very hard time to analyze this fabricated anti-societal world.

Although a number of economic models have been used to interpret exchange rate movements, virtually none of the existing models can explain exchange rate behavior well because it is so much speculation in this market and it makes economic theories useless. Some economists attempt to interpret the phenomenon of deviation of the actual currency values from their fundamental values as speculative bubbles. Particularly, economic agents form their exchange rate expectations based on a certain kind of extrapolative behavior. Thus, favorable changes in financial variables or in the investment environment may tend to generate an exchange rate appreciation that, in turn, may lead to expectations of a further appreciation. But, here, especially with the “euro”, we did not have any changes in fundamentals. The above process continues as long as the market believes the currency price will persist moving in the same direction. Since the actual price moves farther away from the fundamentals as time passes, capital gains would have to be sufficiently large to compensate the risk of a bursting bubble, which it is not obvious for the euro at this moment.

Speculations and speculative bubbles have gained some empirical support in exchange rate determination literature. There were found in the DM/$ and FF/$ rates for the period June to October 1978. Evidence indicates that the German mark was overvalued with respect to its fundamental value by 12% and French franc by 11%. A speculative bubble was also found in the United States, where the dollar appreciated substantially for the period 1980 through 1985. The same seems to be the case with the Euro-zone; euro has been appreciated without any changes in the fundamentals (except the Iraqi war and the fear of another war) since the begging of 2003 reached 1.3646 $/euro on December 30, 2004 and continues to be overvalued. On October 6, 2005 it was 1.2129 $/euro.

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11 Euro was 0.8813 $/euro on December 31, 2001 and has been appreciated by 54.84% in three years. See, Kallianiotis (2005a).

2. Portfolio Diversification and the Euro Shortage

This analysis is including portfolio balance and its implications for exchange rates. A starting point is the hypothesis that real money demand depends not only on income, the conventional transactions variable, but also on interest rate and on wealth, the speculative demand. The internationalization of business and investment opportunities induce speculators to diversify their portfolios of assets denominated in a variety of currencies so that they can maximize their wealth and many times, we have experienced drastic effects on the value of currencies because these speculators decided to change overnight the content of their portfolios.13

These shifts in wealth induced by current account imbalances or portfolio diversification create monetary imbalances leading to adjustments in long-run price level expectations and thus to exchange rate movements. With perfect mobility of capital, these specifications of money demand imply that the real money demand of a country with a surplus or acquiring its assets rises while it falls abroad. The relative price level of the country with a surplus or with a high demand of its assets declines and, therefore, exchange rates for given terms of trade tend to appreciate.

The demand for monies is affected by an international redistribution of wealth. Portfolio effects can arise in the context of imperfect asset substitutability. With uncertain real returns, portfolio diversification makes assets imperfect substitutes and gives rise to determinate demands for the respective securities and to real yield differentials or a higher risk premium that one currency offers relative to the others.

A portfolio model could provide an explanation of the unanticipated euro appreciation that is only poorly accounted for by speculation, high risk of holding U.S. dollar assets,14 and future uncertainty. The system of flexible exchange, the macroeconomic policies, and the lately disturbances15 have created an incentive for portfolio diversification, that the euro will occupy a large share in an efficiently diversified portfolio, and that the resulting portfolio shifts or capital flows account for some of the unanticipated appreciation of this new currency and not the EMU fundamentals.

We would like to measure the real returns on assets denominated in four different currencies, here. The nominal short-term interest rate must be as follows (with ex ante and ex post calculation), depending whether the currency is at a forward discount or at a forward premium:

13 In June 1997, the Asian currency crises started. The Thai baht devaluated in July, followed soon after by the Indonesian rupiah, Korean won, Malaysian ringgit, and Philippine peso. Following these initial exchange rate devaluations, Asian economies plummeted into recessions. The Indonesian president went public and blamed speculators (he named even one, George Soros) who shifted their short-term investments out of the country. Next day this poor president was forced to reign. See, Eiteman, Stonehill, and Moffett (2004, p. 30), Rajan and Zingales (1998), and Singal (1999).


These equations can be expanded as,

\[ i_{S-T_t} = r_t + (p_{t+1} - p_t) + (f_t - s_t) \]  

or  

\[ i_{S-T_t} = r_t + (p_{t+1} - p_t) - (f_t - s_t) \]  

and by lagging prices and exchange rates one period (avoiding their forecasting), we have an ex post measure of the nominal rate of return of an asset,

\[ i_{S-T_t} = r_t + (p_t - p_{t-1}) + (s_t - s_{t-1}) \]  

or  

\[ i_{S-T_t} = r_t + (p_t - p_{t-1}) - (s_t - s_{t-1}) \]  

Then, the ex ante and the ex post real returns will be:

\[ r_t = i_{S-T_t} - \pi_t - fd_t^e \]  

or  

\[ r_t = i_{S-T_t} - \pi_t + fp_t^e \]  

and

\[ r_t = i_{S-T_t} - (p_t - p_{t-1}) - (s_t - s_{t-1}) \]  

\[ r_t = i_{S-T_t} - (p_t - p_{t-1}) + (s_t - s_{t-1}) \]  

where, \( i_{S-T_t} \)=the nominal short-term interest rate (return), \( r \)= the real rate of interest, \( \pi \)= the inflation rate, \( fd \)= the forward discount of the currency, \( fp \)= the forward premium, \( p \)= the \( \ln \) of price index, \( s \)= the \( \ln \) of spot exchange rate, \( f \)= the \( \ln \) of forward exchange rate, and \( \pi^e \) the expected value of the variable.

Now, we take the utility function of an investor who wants to maximize his end-of-period real wealth (\( w \)) by investing on home (\( r \)) and foreign (\( r^* \)) securities and to determine the optimal portfolio share of foreign securities (x).

\[ \text{Max } U = u(\bar{w}, \sigma_w^2) \]  

where, \( U \)= the utility function, \( \bar{w} \)= the mean of the end-of-period random wealth, and \( \sigma_w^2 \)= the variance of wealth, \( r \)= the real return on home securities, \( r^*_x \)= the real return of different foreign
securities, and \( x \) = the optimal portfolio share (weights) on different currencies denominated securities.

The solution of eq. (11) will be to construct a portfolio of four different assets, which will maximize its return, \( E(R_P) \), and minimize its risk, \( \sigma_{R_P}^2 \).

\[
E(R_P) = x_S \cdot r_S + x_{euro} \cdot r_{euro} + x_{pound} \cdot r_{pound} + x_{yen} \cdot r_{yen}
\]

(12)

The Variance Covariance Matrix

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x_S )</th>
<th>( x_{euro} )</th>
<th>( x_{pound} )</th>
<th>( x_{yen} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>( r_S )</td>
<td>( r_{euro} )</td>
<td>( r_{pound} )</td>
<td>( r_{yen} )</td>
</tr>
<tr>
<td>( \sigma_{r_S}^2 )</td>
<td>( \sigma_{r_{euro}}^2 )</td>
<td>( \sigma_{r_{pound}}^2 )</td>
<td>( \sigma_{r_{yen}}^2 )</td>
<td></td>
</tr>
<tr>
<td>( \text{Cov}(r_S, r_{euro}) )</td>
<td>( \text{Cov}(r_S, r_{pound}) )</td>
<td>( \text{Cov}(r_S, r_{yen}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Cov}(r_{euro}, r_{pound}) )</td>
<td>( \text{Cov}(r_{euro}, r_{yen}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Cov}(r_{pound}, r_{yen}) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Then,

\[
\sigma_{R_P}^2 = x_S^2 \sigma_{r_S}^2 + x_{euro}^2 \sigma_{r_{euro}}^2 + x_{pound}^2 \sigma_{r_{pound}}^2 + x_{yen}^2 \sigma_{r_{yen}}^2
+ 2x_S x_{euro} \text{Cov}(r_S, r_{euro}) + 2x_S x_{pound} \text{Cov}(r_S, r_{pound})
+ 2x_S x_{yen} \text{Cov}(r_S, r_{yen})
+ 2x_{euro} x_{pound} \text{Cov}(r_{euro}, r_{pound})
+ 2x_{euro} x_{yen} \text{Cov}(r_{euro}, r_{yen})
+ 2x_{pound} x_{yen} \text{Cov}(r_{pound}, r_{yen})
\]

(13)

Standard Deviations and Correlation Coefficients

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x_S )</th>
<th>( x_{euro} )</th>
<th>( x_{pound} )</th>
<th>( x_{yen} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>( r_S )</td>
<td>( r_{euro} )</td>
<td>( r_{pound} )</td>
<td>( r_{yen} )</td>
</tr>
<tr>
<td>( \sigma_{r_S} )</td>
<td>( \rho_{r_S,r_{euro}} )</td>
<td>( \rho_{r_S,r_{pound}} )</td>
<td>( \rho_{r_S,r_{yen}} )</td>
<td></td>
</tr>
<tr>
<td>( \rho_{r_{euro}} )</td>
<td>( \sigma_{r_{euro}} )</td>
<td>( \rho_{r_{euro},r_{pound}} )</td>
<td>( \rho_{r_{euro},r_{yen}} )</td>
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<td>( \rho_{r_{pound}} )</td>
<td>( \rho_{r_{pound}} )</td>
<td>( \sigma_{r_{pound}} )</td>
<td>( \rho_{r_{pound},r_{yen}} )</td>
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<td>( \rho_{r_{yen}} )</td>
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<td>( \rho_{r_{yen}} )</td>
<td>( \sigma_{r_{yen}} )</td>
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</tr>
</tbody>
</table>

or
The next step is to determine the optimal portfolio share (weights), $x$, $x_{\text{euro}}$, $x_{\text{pound}}$, and $x_{\text{yen}}$.

### 3. The Basic Model of Optimal Portfolio Selection

The basic model of portfolio selection for a risk-averse person is as follows:

Suppose the utility function for a risk-averse person is

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where, $E(R)$ is the expected return of the asset or a portfolio of assets, $A$ is the coefficient of risk aversion of an individual (for risk-averse persons, $A$ is positive; typically, it is 1, or 2), and $\sigma^2$ is the variance of the asset.

For a four-security portfolio, the utility function becomes

$$U = \sum_{i=1}^{4} w_i E(R_i) - \frac{1}{2}A \sum_{i=1}^{4} \sum_{j=1}^{4} w_i w_j \text{cov}(i,j)$$

The constraints are

$$\sum_{i=1}^{4} w_i = 1$$

$$w_i \geq 0, \text{ for } i = 1 \ldots 4$$

For the time being, we will relax the second constraint (18) and allow negative weights of assets. Incorporating the constraint that the sum of the weights is one, the expected return of a four-security portfolio is

$$E(R_p) = w_1 R_1 + w_2 R_2 + w_3 R_3 + (1 - w_1 - w_2 - w_3) R_4$$

Similarly, the variance of a four-security portfolio is

$$\text{var}(R_p) = w_1^2 \text{var}_1 + w_2^2 \text{var}_2 + w_3^2 \text{var}_3 + (1 - w_1 - w_2 - w_3)^2 \text{var}_4 + 2w_1 w_2 \text{cov}_{12}$$

$$+ 2w_1 w_3 \text{cov}_{13} + 2w_1 (1 - w_1 - w_2 - w_3) \text{cov}_{14} + 2w_2 w_3 \text{cov}_{23} + 2w_2 (1 - w_1 - w_2 - w_3) \text{cov}_{24}$$
+ 2w_3(1 - w_1 - w_2 - w_3)\text{cov}_{34} \tag{20}

The utility function becomes

\[ U = w_1R_1 + w_2R_2 + w_3R_3 + (1 - w_1 - w_2 - w_3)R_4 \]

\[- \frac{1}{2}A[w_1^2\text{var}_1 + w_2^2\text{var}_2 + w_3^2\text{var}_3 + (1 - w_1 - w_2 - w_3)^2\text{var}_4 + 2w_1w_2\text{cov}_{12} + 2w_1w_3\text{cov}_{13} + 2w_1(1 - w_1 - w_2 - w_3)\text{cov}_{14} + 2w_2w_3\text{cov}_{23} + 2w_2(1 - w_1 - w_2 - w_3)\text{cov}_{24} + 2w_3(1 - w_1 - w_2 - w_3)\text{cov}_{34}] \tag{21}\]

We set the derivatives of the utility function with respect to the three weights, \(w_1\), \(w_2\), and \(w_3\), to be zero. This gives us

\[
\frac{\partial U}{\partial w_1} = R_1 - R_4 + [\text{var}_4 - \text{cov}_{14} + (2 \text{cov}_{14} - \text{var}_1 - \text{var}_4)w_1 + (\text{cov}_{14} + \text{cov}_{24} - \text{cov}_{12} - \text{var}_4)w_2 + (\text{cov}_{34} + \text{cov}_{14} - \text{var}_4 - \text{cov}_{13})w_3] A = 0 \tag{22}
\]

\[
\frac{\partial U}{\partial w_2} = R_2 - R_4 + [\text{var}_4 - \text{cov}_{24} + (\text{cov}_{14} + \text{cov}_{24} - \text{var}_4 - \text{cov}_{12})w_1 + (2 \text{cov}_{24} - \text{var}_4 - \text{var}_2)w_2 + (\text{cov}_{24} + \text{cov}_{34} - \text{var}_3 - \text{cov}_{23})w_3] A = 0 \tag{23}
\]

\[
\frac{\partial U}{\partial w_3} = R_3 - R_4 + [\text{var}_4 - \text{cov}_{34} + (\text{cov}_{14} + \text{cov}_{34} - \text{var}_4 - \text{cov}_{13})w_1 + (\text{cov}_{24} + \text{cov}_{34} - \text{var}_4 - \text{cov}_{23})w_2 + (2 \text{cov}_{34} - \text{var}_3 - \text{var}_4)w_3] A = 0 \tag{24}
\]

Solving equations (22-24) for \(w_1, w_2, w_3\), we get

\[ w_1 = \alpha/\delta, \quad w_2 = \beta/\delta, \quad w_3 = \gamma/\delta \tag{25} \]

where

\[
\alpha = \{2R_4 - R_2 - R_3 + A(\text{cov}_{23} - \text{cov}_{24} - \text{cov}_{34})\}\text{cov}_{23} + (R_2 - R_3)\text{cov}_{24} + (R_3 - R_2)\text{cov}_{34}\text{cov}_{14} + [(R_4 - R_2)\text{cov}_{34} + (R_2 - R_4 - A \text{cov}_{24})\text{cov}_{23} + (A \text{cov}_{24} + 2R_3 - A \text{cov}_{34} - R_4 - R_2)\text{cov}_{24}] \text{cov}_{13} + [(R_3 - R_4 - A \text{cov}_{34}) \text{cov}_{23} + (-R_4 + A \text{cov}_{34} + 2R_2 - R_3) \text{cov}_{34} + (-A \text{cov}_{34} + R_4 - R_3) \text{cov}_{24}] \text{cov}_{12} + (-R_1 + R_2) \text{cov}_{34}^2 + [(2R_1 - R_4 - R_2) \text{cov}_{34} + (2A \text{cov}_{34} - R_3 - R_4 + 2R_1) \text{cov}_{24} + (R_4 - R_1) \text{cov}_{23}] \text{cov}_{23}
\]
\begin{align}
+ [(R_2 + R_3 - 2R_1 - A \ cov_{23}) \ cov_{23} + (R_2 + A \ cov_{23} - R_3) \ cov_{13} + (R_3 - R_2 + A \ cov_{23}) \ cov_{12}] \ var_4 \\
+ [(2R_1 - R_2 - R_3) \ cov_{34} + (R_3 - R_1) \ cov_{24}] \ cov_{24} \\
+ [(R_2 + A \ cov_{24} - R_4) \ cov_{14} + (-R_2 + A \ cov_{24} + R_4) \ cov_{12} \\
+ (R_2 - 2R_1 + R_4 - A \ cov_{24}) \ cov_{24} + (-R_2 + R_1 - A \ cov_{12}) \ var_4] \ var_3 \\
+ [(R_4 - R_3 + A \ cov_{34}) \ cov_{13} + (R_4 + R_3 - 2R_1 - A \ cov_{34}) \ cov_{34} + (R_3 - R_4 + A \ cov_{34}) \ cov_{14} \\
+ (R_1 - A \ cov_{13} - R_3) \ var_4 + (R_1 - R_4 + A \ var_4 - A \ cov_{14}) \ var_3] \ var_2 \quad (26) \\
\beta = [(R_1 - R_2) \ cov_{34} + (R_4 - R_1) \ cov_{23} + (R_3 - R_1) \ cov_{24}] \ cov_{34} \\
+ [(R_1 - R_3) \ cov_{23} + (R_3 + A \ cov_{23} + R_1 - A \ cov_{13} - 2R_2) \ cov_{13} + (R_3 - R_1 + A \ cov_{13}) \ cov_{12}] \ var_4 \\
+ [(R_1 - R_3) \ cov_{24} + (2R_2 - R_3 - R_1) \ cov_{34} + (2R_3 - R_1 - R_4 - A \ cov_{34}) \ cov_{23} + (A \ cov_{23} + R_3 - R_2) \ cov_{14}] \ cov_{14} \\
+ [(2R_1 - R_4 - R_3 + A \ cov_{34}) \ cov_{13} + (R_3 - R_4 - A \ cov_{34}) \ cov_{13} + (R_4 - R_3 - A \ cov_{34}) \ cov_{14}] \ cov_{12} \\
+ [(R_1 - R_4) \ cov_{23} + (2R_2 - R_3 - R_4 - A \ cov_{23} - A \ cov_{24} + 2A \ cov_{34}) \ cov_{14} \\
+ (2R_2 - R_4 - R_1) \ cov_{34} + (-R_3 - R_1 + 2R_4 - A \ cov_{34}) \ cov_{24} + (R_4 + A \ cov_{24} - R_2) \ cov_{13}] \ cov_{13} \\
+ [(R_1 - 2R_2 + R_4 + A \ cov_{24} - A \ cov_{14}) \ cov_{14} + (A \ cov_{14} + R_4 - R_1) \ cov_{12} \\
+ (R_2 - R_1 - A \ cov_{12}) \ var_4 + (R_1 - R_4) \ cov_{24}] \ var_3 \\
+ [(R_3 + R_4 - 2R_2 - A \ cov_{23}) \ cov_{34} + (R_3 - R_4 + A \ cov_{34}) \ cov_{24} \\
+ (R_4 - R_3 + A \ cov_{34}) \ cov_{23} + (R_2 - R_3 - A \ cov_{23}) \ var_4 \\
+ (R_2 - R_4 - A \ cov_{24} + A \ var_4) \ var_3] \ var_1 \quad (27) \\
\gamma = [(R_2 - R_3 + A \ cov_{23}) \ cov_{14} + (2R_3 - R_2 - R_1) \ cov_{24} + (R_1 - R_2) \ cov_{34} \\
+ (2R_2 - R_1 - R_4 - A \ cov_{24}) \ cov_{23}] \ cov_{14} \\
+ [(R_2 - R_1) \ cov_{34} + (R_1 - R_3) \ cov_{24}] \ cov_{24} + [(R_1 - R_4) \ cov_{34} + (A \ cov_{14} + R_4 - R_1) \ cov_{13} \\
+ (R_1 + R_4 - 2R_3 + A \ cov_{34} - A \ cov_{14}) \ cov_{14} + (R_3 - R_1 - A \ cov_{13}) \ var_4] \ var_2 \\
+ [(R_1 - R_4) \ cov_{23} + (2R_3 - R_1 - R_4 - A \ cov_{34}) \ cov_{24} \\
+ (-R_4 - R_2 + 2A \ cov_{24} + 2R_3 - A \ cov_{34} - A \ cov_{23}) \ cov_{14}]
\end{align}
\[\begin{align*} &+ (-R_1 - R_2 + 2R_4) \text{cov}_{34} + (-A \text{cov}_{24} - R_4 + R_2) \text{cov}_{13} + (-R_3 + R_4 + A \text{cov}_{34}) \text{cov}_{12} \text{cov}_{12} \\
\&+ [(-R_4 + 2R_1 - R_2 + A \text{cov}_{24}) \text{cov}_{24} + (-A \text{cov}_{24} - R_2 + R_4) \text{cov}_{14}] \text{cov}_{13} \\
\&+ (R_4 - R_1) \text{cov}_{24} \text{cov}_{23} \\
\&+ [(A \text{cov}_{23} + R_2 + R_1 + A \text{cov}_{13} - 2R_3 - A \text{cov}_{12}) \text{cov}_{12} + (-R_2 + R_1) \text{cov}_{23}] \\
\&+ (-R_1 + R_2) \text{cov}_{13} \text{var}_{4} \\
\&+ [(-A \text{cov}_{24} + R_4 + R_2 + A \text{cov}_{34} - 2R_3) \text{cov}_{24} + (-R_4 + R_2) \text{cov}_{34}] \\
\&+ (-R_2 + R_3 - A \text{cov}_{23}) \text{var}_{4} + (R_4 + A \text{cov}_{24} - R_2) \text{cov}_{23} \\
\&\quad+ (R_3 + A \text{var}_{4} - R_4 - A \text{cov}_{34}) \text{var}_{2} \text{var}_{1} \quad (28) \\
\delta &= A \{[\text{var}_1 \text{var}_2 + (\text{var}_2 + \text{var}_1)\text{var}_3]\text{var}_4 + \text{var}_1 \text{var}_2 \text{var}_3 \\
\&+ [2\text{cov}_{12} \text{var}_4 - 2\text{var}_1 \text{var}_4 + 2(\text{var}_4 + \text{cov}_{12})\text{cov}_{13} + (2\text{cov}_{14} - \text{var}_4 - \text{var}_1)\text{cov}_{23} \\
\&+ (2\text{cov}_{14} - 2\text{cov}_{13} - 2\text{cov}_{12})\text{cov}_{14}]\text{cov}_{23} \\
\&+ [2\text{cov}_{12} \text{var}_3 - 2\text{var}_2 \text{var}_3 + 2\text{var}_2 \text{cov}_{13} - (\text{var}_2 + \text{var}_3)\text{cov}_{14}]\text{cov}_{14} \\
\&+ [2\text{cov}_{12} \text{var}_4 - (\text{var}_4 + \text{var}_2)\text{cov}_{13} - 2\text{var}_2 \text{var}_4]\text{cov}_{13} \\
\&\quad- [2\text{var}_4 \text{var}_3 + (\text{var}_3 + \text{var}_4)\text{cov}_{12}]\text{cov}_{12} \\
\&+ [2(\text{cov}_{12} - \text{var}_1)\text{var}_3 + 2(\text{cov}_{13} - \text{cov}_{12})\text{cov}_{13} + 2(\text{var}_3 - \text{cov}_{13} + \text{cov}_{12})\text{cov}_{14} \\
\&+ (2\text{cov}_{13} - \text{var}_3 - \text{var}_1)\text{cov}_{24} + 2(\text{var}_1 - \text{cov}_{13} - \text{cov}_{14})\text{cov}_{23}]\text{cov}_{24} \\
\&+ [2(\text{var}_2 - \text{cov}_{12})\text{cov}_{13} - 2\text{var}_1 \text{var}_2 + 2\text{cov}_{12}^2 + 2(\text{cov}_{23} + \text{var}_1 - \text{cov}_{12} - \text{cov}_{13})\text{cov}_{24} + 2(\text{var}_1 - \text{cov}_{14} - \text{cov}_{12})\text{cov}_{23} + 2(\text{cov}_{13} - \text{cov}_{12} + \text{var}_2)\text{cov}_{14} + (2\text{cov}_{12} - \text{var}_2 - \text{var}_1)\text{cov}_{34} \text{cov}_{34}] \quad (29) \end{align*}\]

Suppose the variance-covariance matrix for four assets is

\[
\begin{pmatrix}
0.04 & 0.03 & 0.04 & 0.05 \\
0.03 & 0.08 & 0.05 & 0.06 \\
0.04 & 0.05 & 0.12 & 0.07 \\
0.05 & 0.06 & 0.07 & 0.16
\end{pmatrix}
\]

Then \(\text{var}_1 = 0.04, \text{var}_2 = 0.08, \text{var}_3 = 0.12, \text{var}_4 = 0.16, \text{cov}_{12} = 0.03, \text{cov}_{13} = 0.04, \text{cov}_{14} = 0.05, \text{cov}_{23} = 0.05, \text{cov}_{24} = 0.06, \text{cov}_{34} = 0.07.\)
(1) Assume that the coefficient of risk aversion, \( A = 1 \). Further, assume that the expected returns from the four assets are: \( R_1 = .10, R_2 = .12, R_3 = .14, R_4 = .16 \)

Substituting these numbers, we find, the optimal weights to be:

\[
w_1 = .02, \ w_2 = .26, \ w_3 = .34, \ w_4 = .38.\]

The expected return of the portfolio is

\[
E(R_p) = .02 \times 1 + .26 \times 1.12 + .34 \times 1.14 + .36 \times 1.16 = .1384
\]

The variance of returns of the portfolio is

\[
\text{var}(R_p) = 0.0828, \text{ or } \sigma(R_p) = .2877.
\]

The maximum value of the utility function is

\[
\text{Max } U = .1002
\]

If we select only the first asset, the utility is \(.1 - .5 \times .04 = .08\). Similarly, if we choose only the second, or third, or fourth asset, the utility in each case is .08. If we distribute the money evenly in the four assets, that is \(w_1 = w_2 = w_3 = w_4 = .25\), then the utility is \(U = .09875\).

(2) Assume that the coefficient of risk aversion, \( A = 0.5 \). If a person is willing to take more risk, his A is possibly .5. The weights of the portfolio in this case are \(w_1 = -.9000, w_2 = .3000, w_3 = .7, \text{ and } w_4 = .9\).

(3) Assume that the coefficient of risk aversion, \( A = 1.5 \). For a person, who is more risk-averse, the coefficient \( A = 1.5 \). For him the optimal weights are \(w_1 = .3267, w_2 = .2467, w_3 = .2200, w_4 = .2067\).

For less risk-averse individuals, the fourth weight will get higher at the expense of the first weight. The critical \( A \), where the first weight becomes zero is \( .9787234043 \). With the constraint of all positive weights, we can solve the problem for persons with \( A > .9787 \).

4. Assets Risk and Return Revisited

We assume that a spot exchange rate \( S_t \) follows a random walk,

\[
S_{t+1} = S_t + \epsilon_{t+1}
\]  \hspace{1cm} (29)

where, \( \epsilon_{t+1} \sim N(0, \sigma_i^2) \). The variance of the error term depends on \( t \), and the objective of the model is to characterize the way, in which this variance changes over time. This volatility of the exchange rate measures the fd or fp of a currency and affects the real return of the assets.
denominated in this specific currency.

In order to measure the exchange rate risk, we use Bollerslev’s (1986) model, which is an extension of Engle’s (1982) original work by developing a technique that allows the conditional heteroscedastic variance to be an ARMA process. This process is the Generalized ARCH (p, q), called the GARCH (p, q), in which the variance is given by

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j e_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

(30)

where, $e_t$ = the disturbances or estimated residuals and $\sigma_t^2 =$ the variance of $\{e_t\}$.

When $\alpha + \beta < 1$, the variance process displays mean reversion to the unconditional expectation of $\sigma_t^2$, $\omega / (1 - \alpha - \beta)$. That is, forecasts of volatility in the distant future will be equal to the unconditional expectation of $\sigma_t^2$, $\omega / (1 - \alpha - \beta)$. The GARCH model has been used to characterize patterns of volatility in U.S. dollar foreign exchange markets\textasciitilde16 and in the European Monetary System\textasciitilde17.

Meese and Rogoff (1983) conclude that exchange rate models do a poor job of tracking movements over short horizons. Then, the macroeconomic variables (money supply, income, interest rate, price level, debt, etc.) can explain changes in exchange rate over medium and long horizons. Currency traders, speculators, and other market participants who focus on the short-term horizon look beyond macroeconomic models. They, search for signs (like risk and return) of short-term changes in the demand for currencies (assets denominated in specific currency), using any available measures of market transactions, behavior, and news. It is important for economists to model short-term exchange rate dynamics and determine (forecast) the future value of the different currencies. Speculators in the future market are constantly interpreting public and private information about ongoing shifts in foreign currency demand as they develop their directional views\textasciitilde18.

The first step in evaluating the strength of any relationship between real rate of return and exchange rates is to look for visual evidence. Plotting the levels of the real rate of return against exchange rate levels reveals no obvious patterns. However, a fairly clear relationship emerges when looking at changes in the two variables. Knowing the change of the real rate of return of a country would have allowed someone to guess correctly the direction of the $\$, euro, pound, and yen.

We can run regressions to see the effects of real return and risk on the exchange rate, too.

$$s_i = a_0 + a_1 r_i + a_2 \sigma_r^2 + \epsilon_t$$

(31)

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\textsuperscript{17} See, Neely (1999).
\textsuperscript{18} See, Klitgaard and Weir (2004).
where, $s_t$=the spot exchange rate, $r_t$=the real rate of return, and $\sigma_r^2$=the variance (risk) of the real rate of interest, calculating with the use of eq. (30).

A similar equation can be run by testing the real rate of interest as a function of spot exchange rate and risk.

$$r_t = \beta_0 + \beta_1 s_t + \beta_2 \sigma_r^2 + \epsilon_t$$  \hspace{1cm} (32)

Furthermore, tests show that movements of rate of return and its risk in one country anticipate how speculators change their demand and supply of assets denominated in this specific currency. The nature of exchange rate dynamics could argue about the contemporaneous relationship between return/risk and exchange rates and their future trends.

Currency market participants are heterogeneous and act on their own bits of private information, as well as on public information.\(^\text{19}\) Examples of private information include participants’ expectations of future economic variables, perceptions of official and private sector demand, and perceptions of developing shifts in global liquidity and risk taking. Speculators act immediately in advance of exchange rate movements in a way that anticipates the direction of exchange rates and the rate of return.

Our objective is to seek data to help us understand what is driving the exchange rate at any given time. Variables that are viewed as fundamental to dictating currency values (relative money supply, output, inflation rates, interest rate differentials, etc.) are constantly analyzed and forecast. Various transaction data are also examined to determine demand changes in different currencies. The results suggest that real rate of return and risk in different countries merit inclusion in policy analysis and in ongoing research on exchange rate dynamics and determination.

5. Some Preliminary Empirical Results

So far, we have discussed the theoretical part of the rate of return of an asset denominated in foreign currency, the rate of return of a portfolio of currencies and the measurement of risk of the individual currency, due to unanticipated exchange rate movements, and the calculation of portfolio risk. The data, taken from economagic.com and imfstatistics.org, are monthly from 1973:03 to 2005:08. They comprise spot exchange rate, money supply (M2), consumer price index (CPI), federal funds rate, 3-month T-bill rate, prime rate, government bonds rate, real GDP, current account, unemployment rate, budget deficit, national debt, personal saving rate, price of gold, price of oil, and stock market index (DJIA).

Table 1 presents the mean values and standard deviations of different variables (spot exchange rate, forward discount or forward premium, real rate of return, short-term interest rates, inflation rates, etc.). Also, it gives correlation coefficient between these variables. The sample is divided into three periods 1973:03-2005:08, 1973:03-2001:12, and 2002:01-2005:08. The U.S. dollar was at a premium with respect to euro, pound, and yen until 2001:12 and it is at a discount

\(^{19}\) See, Evans and Lyons (2002).
after 2002:01 to present. The real rate of return for a European investing in the U.S. is negative; the same is true for a British. For a Japanese investor to invest in the U.S., the return is positive. Then, we do not expect an increase in demand for U.S. financial assets and there will be no appreciation of the U.S. dollar, due to these negative real returns.

Table 2 shows the mean values and standard deviations of the different economic fundamentals. Some variables that have changed drastically in the U.S. are the short-term interest rates (very low) and the negative real risk free rate of interest; also, the high risk (RP=3.43), the current account deficit, the budget deficit, national debt, low savings, high price of gold and oil.

Table 3 gives the correlation between the exchange rates and the fundamentals. The ($/euro) exchange rate is highly correlated with federal funds rate, T-Bill rate, and prime rate; also, with the risk premium, the unemployment rate, and the budget deficit. From 2002:01 to now the correlations are small; then, the ($/euro) exchange rate does not depend on fundamentals. The same holds for the other two exchange rates ($/pound) and (yen/$).

Table 4 provides the causality tests between the economic fundamentals in the U.S.A. and the three different exchange rates. From 1973:03 to 2001:12, the inflation rate and the real risk free rate of interest caused the ($/euro) exchange rate; also, the money supply, the real GDP growth, the unemployment rate, and the DJIA caused the ($/pound) rate; and the risk premium and the budget deficit caused the (yen/$) exchange rate. From 2002:01 to present, the ($/euro) exchange rate does not depend on any fundamentals (just speculation); the ($/pound) exchange rate is caused by the personal saving rate; and the (yen/$) rate is caused by the price of gold.

Tables 5a, 5b, and 5c give the variance-covariance and the standard deviation and correlation coefficients matrices for four different real rates of interest. The real return that an American receives by investing in U.S. T-Bills (rA), the real return that a European makes by investing in the same instrument (reU), the real return that a Briton is making for the same investment (rB), and the real return that a Japanese is receiving by investing in U.S. T-Bills (rJ). The real return is high from 1999:01 to 2001:12 (from 2.177% to 11.187%) and the risk very high, too (from 3.107% to 31.392%). This high real return on U.S. financial assets has caused the dollar to appreciate (from 4.076% to 9.011% p.a.). After the introduction of the Euro-notes (2002:01), the real return on the U.S. financial assets is becoming negative (-1.200% to -9.598%) and the risk stayed at the same level as before (from 3.415% to 30.022%). These negative returns have caused the dollar to depreciate (from 3.649% to 8.398% p.a.). Then, speculators are seeking higher returns outside of the U.S. and this capital outflows have depreciated the U.S. dollar. The highest appreciation is in euro, follows by pound, and lastly, the yen. Finally, Table 6 shows that the high returns before 2002 have been caused by the high real return in the U.S., the high inflation, and the high growth in the value of the dollar. The low returns after 2002 have been caused, due to a decline in real returns for Europeans and Britons, reductions in T-bill rate, and depreciation of the U.S. dollar with respect the euro.

Of course, data from Euro-zone, U.K., and Japan are needed to test the effects of those countries fundamentals on the three exchange rates, which is an analysis that will be done in the future.
6. Policy Implications

Even though that the U.S. dollar has depreciated drastically since 2001 (i.e., -52.66% with respect to euro),\textsuperscript{20} the current account deficits have assumed extraordinary proportions. A current account deficit is matched by a capital account surplus. In other words, a country with a current account deficit surrenders claims on future income (physical assets, stocks, and bonds) to foreigners. The ongoing U.S. current account deficit translates into an average of billions dollars in net capital imports per business day. That is, foreign investors have been accumulating U.S. assets at an unusually high rate. Foreign investors might become wary of holding increasingly larger portions of their wealth in U.S. assets. In order to promote continued investment in the United States, U.S. assets would then have to become more attractive. One way of attracting foreign investments is to lower the price of the asset in foreign currency terms. A decline in the foreign exchange value (depreciation) of the dollar would do just that. Therefore, a large current account deficit might be expected to depress the value of the dollar over time.

A reasonable question arises now; but, what about the persistent current account deficit? Indispensably, trade policies must improve it and citizens must make their demands for imports more elastic $(\varepsilon_{\text{M}} > 1)$ for their own good and their country’s benefits. The following identity holds for an economy,

$$Y - E = T - G + S - I = X - M$$

where, $Y$=income (GDP), $E$=expenditures, $T$=taxes, $G$=government spending, $S$=saving, $I$=investment, $X$=exports, and $M$=imports.

If $(X - M < 0)$ in the above eq. (33), a devaluation might improve this current account deficit. But, a necessary and sufficient condition (Marshall-Lerner) must hold,

$$|\varepsilon_{\text{M}}| + |\varepsilon_{\text{M}^*}| > 1$$

where, $\varepsilon_{\text{M}}$ =the domestic price elasticity of the demand for imports and $\varepsilon_{\text{M}^*}$ =the foreign price elasticity of demand for their imports.

Then, the process could be as follows (if Marshall-Lerner condition holds):

$$CAD \uparrow \Rightarrow (KAS) \uparrow \Rightarrow P_{\text{assets}} \downarrow \Rightarrow (S) \uparrow \Rightarrow CAD \downarrow$$

where, CAD=current account deficit and KAS=capital account surplus.

The current account and capital account are two sides of the same coin. A country that is running a current account deficit $(M_{\text{Goods}} > X_{\text{Goods}})$ is necessarily also running a capital account surplus $(X_{\text{Financial Assets}} > M_{\text{Financial Assets}})$. Foreign-owned assets in the United States increased from less than $2.5$ trillion in 1990 to over $10$ trillion by the end of 2003. Over the same period, U.S.-

\textsuperscript{20} See, Kallianiotis (2005b, Table 1).
owned assets abroad increased from $2.3 trillion to nearly $7.9 trillion.  

Even though that the real return is lower in the U.S., investors invest here, because of the unparalleled efficiency, stability, transparency, and liquidity of the U.S. financial markets. Investors find that dollar-denominated claims are an attractive element of any international portfolio. This process of investors seeking the most beneficial combination of risk and return, rebalancing portfolio when opportunities arise, gives rise to a source of capital account dynamics that is unrelated in any direct way to the pattern of trade in goods and services. Figure 1 shows a smoothing of the series (the four real returns in U.S. that foreign investors face) by using the Hodrick-Prescott (1997) filter to obtain a smooth estimate of the long-term components of the series (r’s). The graph reveals that the L-T real returns are increasing for foreign investors investing in the U.S. financial assets; then, this demand for U.S. assets will appreciate the U.S. dollar, too.

7. Concluding Comments

The objective of this analysis is to determine the exchange rates between the U.S. dollar and three other currencies, the euro, the pound, and the yen. Lately, the U.S. dollar is loosing value with respect the other three major currencies of the world and we want to see if this depreciation depends on economic fundamentals (lower real return in the U.S. and higher risk) or it is just speculation from individuals or countries, which hold large amounts of foreign assets denominated in different currencies. The preliminary conclusion is, here, that, due to high risk (wars and political conflicts) and low returns many speculators have invested in euros, pounds, and yens instead in dollars denominated assets. Historically, the U.S. has frozen the foreign assets inside the U.S. when a conflict arises. The L-T smoothing of these returns shows that they are growing, so the demand for U.S. investment will increase and the U.S. is expected to appreciate in the future. Investors know this and speculators take advantage of this knowledge.

Also, by constructing a portfolio of four different assets, we can maximize the utility function of a speculator by maximizing his return and minimizing his risk. From these real returns, we can conclude if the currency will appreciate or not. High expected real return on assets denominated in euros means that euro is expected to appreciate. The preliminary tests show that economic fundamentals have less effect on exchange rates, lately; then, exchange rates depend mostly on speculation, due to the expected risk and return. The paper needs some more date from all the countries involved and more statistical and portfolio analysis to give better results.

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References


