INTEGRATION BEE FINALS

The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

Contestants must circle their final answer. Contestants do **not** need to include the constant of integration $+C$ in their answer.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

READY,
GET SET,...

2:30

$$\int \frac{x^2 + 1}{x^3 - x^2 + x - 1}\, dx$$

$$\int \frac{x^2 + 1}{x^3 - x^2 + x - 1} \, dx$$

$$= \int \frac{x^2 + 1}{(x^2 + 1)(x - 1)} \, dx$$

$$= \int \frac{1}{x - 1} \, dx$$

$$= \boxed{\ln |x - 1| + C}$$

READY,
GET SET,...

2:30

$$\int \frac{(x+1)(x+2)}{x+3} \, \mathrm{d}x$$

$$\int \frac{(x+1)(x+2)}{x+3}\,dx$$

$$= \int \frac{x^2 + 3x + 2}{x+3}\,dx = \int \left(\frac{x(x+3)}{x+3} + \frac{2}{x+3} \right) dx$$

$$= \int \left(x + \frac{2}{x+3} \right) dx \qquad \text{or use long division}$$

$$= \boxed{\frac{x^2}{2} + 2\ln|x+3| + C}$$

READY,
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2:30

$$\int \frac{1}{x + \frac{9}{x^2}} \, dx$$

$$\int \frac{1}{x + \frac{9}{x^2}} \, dx$$

$$= \int \frac{x^2}{x^3 + 9} \, dx$$

$$= \frac{1}{3} \int \frac{1}{u} \, du \qquad u = x^3 + 9, \quad du = 3x^2 \, dx$$

$$= \frac{1}{3} \ln |u| + C$$

$$= \boxed{\frac{1}{3} \ln |x^3 + 9| + C}$$

READY,
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2:30

$$\int \sqrt{x^4 + 5x^2 - 6x + (x+3)^2}\, dx$$

$$\int \sqrt{x^4 + 5x^2 - 6x + (x+3)^2}\, dx$$

$$= \int \sqrt{x^4 + 5x^2 - 6x + x^2 + 6x + 9}\, dx$$

$$= \int \sqrt{x^4 + 6x^2 + 9}\, dx = \int \sqrt{(x^2+3)^2}\, dx$$

$$= \int (x^2 + 3)\, dx$$

$$= \boxed{\frac{1}{3}x^3 + 3x + C}$$

READY,
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2:30

$$\int \left(\frac{1}{x} - \frac{1}{3x^2 + 2x} \right) dx$$

$$\int \left(\frac{1}{x} - \frac{1}{3x^2 + 2x} \right) dx$$

$$= \int \frac{3x + 1}{3x^2 + 2x} dx$$

$$= \frac{1}{2} \int \frac{1}{u} du \qquad u = 3x^2 + 2x, \quad du = (6x + 2)\, dx = 2(3x + 1)\, dx, \quad \text{or partial fractions}$$

$$= \frac{1}{2} \ln |u| + C = \boxed{\frac{1}{2} \ln |3x^2 + 2x| + C}$$

READY,
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2:30

$$\int \frac{1}{\sqrt{x}(x + 2\sqrt{x} + 1)}\,\mathrm{d}x$$

$$\int \frac{1}{\sqrt{x}(x + 2\sqrt{x} + 1)}\, dx$$

$$= \int \frac{1}{\sqrt{x}(\sqrt{x} + 1)^2}\, dx$$

$$= 2\int u^{-2}\, du \qquad u = \sqrt{x} + 1, \quad du = \frac{dx}{2\sqrt{x}}$$

$$= -\frac{2}{u} + C$$

$$= \boxed{-\frac{2}{\sqrt{x} + 1} + C}$$

READY,
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2:30

$$\int \left(\frac{\sec x}{1 + \tan x} \right)^2 \mathrm{d}x$$

$$\int \left(\frac{\sec x}{1 + \tan x} \right)^2 dx$$

$$= \int \frac{1}{u^2} du \qquad u = 1 + \tan x, \quad du = \sec^2 x \, dx$$

$$= -\frac{1}{u} + C$$

$$= \boxed{-\frac{1}{1 + \tan x} + C}$$

READY,
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2:30

$$\int \frac{\ln(\ln x)}{x \ln x} \, dx$$

$$\int \frac{\ln(\ln x)}{x \ln x} \, dx$$

$$= \int u \, du \qquad \textcolor{orange}{u = \ln(\ln x), \quad du = \frac{dx}{x \ln x}}$$

$$= \frac{u^2}{2} + C = \boxed{\frac{1}{2}(\ln(\ln x))^2 + C}$$

READY,
GET SET,...

2:30

$$\int \frac{1 - e^{\cot x}}{1 - \cos^2 x}\, \mathrm{d}x$$

$$\int \frac{1 - e^{\cot x}}{1 - \cos^2 x} \, dx$$

$$= \int \frac{1 - e^{\cot x}}{\sin^2 x} \, dx$$

$$= \int \left(\csc^2 x - \csc^2 x \, e^{\cot x} \right) dx$$

$$= -\cot x + \int e^u \, du \qquad u = \cot x, \quad du = -\csc^2 x \, dx$$

$$= -\cot x + e^u + C = \boxed{-\cot x + e^{\cot x} + C}$$

READY,
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2:30

$$\int \frac{x^3 + 2x^2 + 12x + 18}{x^3(x+3)^2} \, dx$$

$$\int \frac{x^3 + 2x^2 + 12x + 18}{x^3(x+3)^2}\,dx$$

$$= \int \frac{x^3 + 2(x+3)^2}{x^3(x+3)^2}\,dx$$

$$= \int \left(\frac{1}{(x+3)^2} + 2x^{-3}\right)dx$$

$$= \boxed{-\frac{1}{x+3} - \frac{1}{x^2} + C}$$

READY,
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2:30

$$\int \frac{x^2}{\sqrt{x+1}}\, \mathrm{d}x$$

$$\int \frac{x^2}{\sqrt{x+1}}\, dx$$

$$= \int \frac{u^2 - 2u + 1}{\sqrt{u}}\, du \qquad {\color{red}u = x+1, \quad x^2 = (u-1)^2 = u^2 - 2u + 1}$$

$$= \int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + u^{-\frac{1}{2}} \right) du$$

$$= \frac{2}{5} u^{\frac{5}{2}} - \frac{4}{3} u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + C$$

$$= \boxed{\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}} + 2(x+1)^{\frac{1}{2}} + C}$$

READY,
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$$\int e^x \ln e^{5x} \, dx$$

$$\int e^x \ln e^{5x} \, dx$$

$$= \int 5xe^x \, dx \qquad \text{Integration by parts:} \qquad \begin{aligned} u &= x & dv &= e^x \, dx \\ du &= dx & v &= e^x \end{aligned}$$

$$= 5\left(xe^x - \int e^x \, dx \right)$$

$$= \boxed{5\left(xe^x - e^x \right) + C}$$

READY,
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2:30

$$\int \frac{\ln x}{x + x \ln^2 x} \, dx$$

$$\int \frac{\ln x}{x + x \ln^2 x}\, dx$$

$$= \int \frac{\ln x}{x(1 + \ln^2 x)}\, dx$$

$$= \frac{1}{2} \int \frac{1}{u}\, du \qquad u = 1 + \ln^2 x, \quad du = \frac{2\ln x}{x}\, dx$$

$$= \frac{1}{2} \ln |u| + C = \boxed{\frac{1}{2} \ln(1 + \ln^2 x) + C}$$