

INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS



You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, notify a grader before the next integral is displayed.

For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.



INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS



All answers must be **simplified** which means

- Rational numbers must be simplified and reduced.

$$\frac{14}{2} \text{ X vs } 7 \text{ ✓}$$

$$\frac{1}{3} + \frac{1}{6} \text{ X vs } \frac{1}{2} \text{ ✓}$$

$$e^{\ln 2} \text{ X vs } 2 \text{ ✓}$$

$$\ln 1 \text{ X vs } 0 \text{ ✓}$$

- Trigonometric functions must be evaluated when possible.

$$\sin \frac{\pi}{3} \text{ X vs } \frac{\sqrt{3}}{2} \text{ ✓}$$

$$\tan 0 \text{ X vs } 0 \text{ ✓}$$



INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS



- You do **not** need to combine multiple logs or fractions with irrational numbers (e.g. π , e , $\ln 3$).

$$2 \ln 3 \checkmark, \ln 9 \checkmark$$

$$\frac{1}{e} + 2 \checkmark, \frac{1 + 2e}{e} \checkmark$$

- However, **no** negative powers should remain.

$$e^{-1} \times \text{ vs } \frac{1}{e} \checkmark$$

At most five participants will move onto the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary.

Everyone moving onto the finals will receive \$25.

Any questions?

**READY,
GET SET,...**

2:00

INTEGRAL #1

$$\int_2^5 (3x^2 + 2x - 5) dx$$

INTEGRAL #1

$$\int_2^5 (3x^2 + 2x - 5) dx$$

$$= \left[3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} - 5x \right]_2^5$$

$$= \left[x^3 + x^2 - 5x \right]_2^5$$

$$= (125 + 25 - 25) - (8 + 4 - 10)$$

$$= 123$$

**READY,
GET SET,...**

2:00

INTEGRAL #2

$$\int_1^2 \left(\frac{1}{x^3} + 4x \right) dx$$

INTEGRAL #2

$$\int_1^2 \left(\frac{1}{x^3} + 4x \right) dx$$

$$= \int_1^2 (x^{-3} + 4x) dx$$

$$= \left[\frac{x^{-2}}{-2} + 4 \cdot \frac{x^2}{2} \right]_1$$

$$= \left[-\frac{1}{2x^2} + 2x^2 \right]_1$$

$$= \left(-\frac{1}{8} + 8 \right) - \left(-\frac{1}{2} + 2 \right) = \boxed{\frac{51}{8}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #3

$$\int_0^1 x(1-x^2)^{10} dx$$

INTEGRAL #3

$$\int_0^1 x(1-x^2)^{10} dx$$

$$= -\frac{1}{2} \int_1^0 u^{10} du \quad u = 1 - x^2, \quad du = -2x dx$$

$$= \left[-\frac{1}{2} \cdot \frac{u^{11}}{11} \right]_1^0$$

$$= \left[-\frac{u^{11}}{22} \right]_1^0$$

$$= 0 - \left(-\frac{1}{22} \right) = \boxed{\frac{1}{22}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #4

$$\int_0^1 (1 - \sqrt{x})(1 + \sqrt{x}) dx$$

INTEGRAL #4

$$\int_0^1 (1 - \sqrt{x})(1 + \sqrt{x}) dx$$

$$= \int_0^1 (1 - x) dx$$

$$= \left[x - \frac{x^2}{2} \right]_0^1$$

$$= \left(1 - \frac{1}{2} \right) - 0$$

$$= \boxed{\frac{1}{2}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #5

$$\int_0^{(\ln 3)/2} (10e^x)^2 dx$$

INTEGRAL #5

$$\int_0^{(\ln 3)/2} (10e^x)^2 dx$$

$$= \int_0^{(\ln 3)/2} 100e^{2x} dx$$

$$= 50 \int_0^{\ln 3} e^u du \quad u = 2x, \quad du = 2 dx$$

$$= [50e^u]_0^{\ln 3}$$

$$= 50 \cdot 3 - 50 = \boxed{100}$$

**READY,
GET SET,...**

2:00

INTEGRAL #6

$$\int_0^1 \left(\sqrt[4]{x^3} - \sqrt[3]{x^4} \right) dx$$

INTEGRAL #6

$$\int_0^1 (\sqrt[4]{x^3} - \sqrt[3]{x^4}) dx$$

$$= \int_0^1 (x^{3/4} - x^{4/3}) dx$$

$$= \left[\frac{4}{7} \cdot x^{7/4} - \frac{3}{7} \cdot x^{7/3} \right]_0^1$$

$$= \left(\frac{4}{7} - \frac{3}{7} \right) - 0$$

$$= \boxed{\frac{1}{7}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #7

$$\int_{1/\pi}^{3/\pi} \frac{\sin(1/x)}{3x^2} dx$$

INTEGRAL #7

$$\int_{1/\pi}^{3/\pi} \frac{\sin(1/x)}{3x^2} dx$$

$$= -\frac{1}{3} \int_{\pi}^{\pi/3} \sin u \, du \quad u = \frac{1}{x} dx, \quad du = -\frac{1}{x^2} dx$$

$$= \frac{1}{3} [\cos u]_{\pi}^{\pi/3}$$

$$= \frac{1}{3} \left(\frac{1}{2} - (-1) \right)$$

$$= \boxed{\frac{1}{2}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #8

$$\int_{-5}^0 \frac{1}{\sqrt{4-x}} dx$$

INTEGRAL #8

$$\int_{-5}^0 \frac{1}{\sqrt{4-x}} dx$$

$$= - \int_9^4 \frac{1}{\sqrt{u}} du \quad u = 4 - x, \quad du = -dx$$

$$= \int_4^9 u^{-1/2} du$$

$$= \left[2u^{1/2} \right]_4^9$$

$$= 2\sqrt{9} - 2\sqrt{4} = \boxed{2}$$

**READY,
GET SET,...**

2:00

INTEGRAL #9

$$\int_1^{e^2} \frac{(\ln(x^2))^2}{x} dx$$

INTEGRAL #9

$$\int_1^{e^2} \frac{(\ln(x^2))^2}{x} dx$$

$$= \frac{1}{2} \int_0^4 u^2 du \quad u = \ln(x^2) = 2 \ln x, \quad du = \frac{2}{x} dx$$

$$= \left[\frac{1}{2} \cdot \frac{u^3}{3} \right]_0^4$$

$$= \frac{1}{2} \cdot \frac{64}{3} - 0$$

$$= \boxed{\frac{32}{3}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #10

$$\int_0^1 (1 + 2x^3)^2 dx$$

INTEGRAL #10

$$\int_0^1 (1 + 2x^3)^2 dx$$

$$= \int_0^1 (1 + 4x^3 + 4x^6) dx$$

$$= \left[x + 4 \cdot \frac{x^4}{4} + 4 \cdot \frac{x^7}{7} \right]_0^1$$

$$= \left[x + x^4 + \frac{4}{7}x^7 \right]_0^1$$

$$= \left(1 + 1 + \frac{4}{7} \right) - 0 = \boxed{\frac{18}{7}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #11

$$\int_0^2 \frac{6}{3x+1} dx$$

INTEGRAL #11

$$\int_0^2 \frac{6}{3x+1} dx$$

$$= 2 \int_1^7 \frac{1}{u} du \quad u = 3x + 1, \quad du = 3 dx$$

$$= 2 \left[\ln |u| \right]_1^7$$

$$= 2(\ln 7 - \ln 1)$$

$$= 2 \ln 7 \text{ or } \ln 49$$

**READY,
GET SET,...**

2:00

INTEGRAL #12

$$\int_0^2 \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right) (1 - x + x^2 - x^3) dx$$

INTEGRAL #12

$$\int_0^2 \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right) (1 - x + x^2 - x^3) dx$$

$$= \int_0^{-4/3} u du \quad u = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}, \quad du = (1 - x + x^2 - x^3) dx$$

$$= \left[\frac{u^2}{2} \right]_0^{-4/3}$$

$$= \frac{1}{2} \left(-\frac{4}{3} \right)^2 - 0$$

$$= \boxed{\frac{8}{9}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #13

$$\int_0^{\pi/2} x \sin x \, dx$$

INTEGRAL #13

$$\int_0^{\pi/2} x \sin x \, dx$$

$$= \left[-x \cos x \right]_0^{\pi/2} + \int_0^{\pi/2} \cos x \, dx$$

Integration by parts:

$$\begin{aligned} u &= x & dv &= \sin x \, dx \\ du &= dx & v &= -\cos x \end{aligned}$$

$$= \left[-x \cos x + \sin x \right]_0^{\pi/2}$$

$$= (0 + 1) - (0 + 0)$$

$$= \boxed{1}$$

**READY,
GET SET,...**

2:00

INTEGRAL #14

$$\int_1^2 \frac{1}{1 - \frac{x}{x+1}} dx$$

INTEGRAL #14

$$\int_1^2 \frac{1}{1 - \frac{x}{x+1}} dx$$

$$= \int_1^2 (x+1) dx \quad \text{Simplify the integrand}$$

$$= \left[\frac{x^2}{2} + x \right]_1^2$$

$$= \left(\frac{4}{2} + 2 \right) - \left(\frac{1}{2} + 1 \right)$$

$$= \boxed{\frac{5}{2}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #15

$$\int_0^{\pi/3} \frac{\sin x}{\cos^2 x + \sin^2 x + 5} dx$$

INTEGRAL #15

$$\int_0^{\pi/3} \frac{\sin x}{\cos^2 x + \sin^2 x + 5} dx$$

$$= \int_0^{\pi/3} \frac{\sin x}{6} dx \quad \text{Trig identity: } \sin^2 x + \cos^2 x = 1$$

$$= \left[-\frac{1}{6} \cos x \right]_0^{\pi/3}$$

$$= -\frac{1}{6} \cdot \frac{1}{2} + \frac{1}{6} \cdot 1$$

$$= \boxed{\frac{1}{12}}$$