The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

**The last person remaining wins an additional $75** and will be crowned the **Integration Champion**!
INTEGRAL #1

READY,
GET SET,...

2:30
INTEGRAL #1

\[
\int \frac{x}{x^4 + 4x^2 + 4} \, dx
\]

2:30
\[
\int \frac{x}{x^4 + 4x^2 + 4} \, dx
\]
\[
= \int \frac{x}{(x^2 + 2)^2} \, dx
\]
\[
= \frac{1}{2} \int \frac{1}{u^2} \, du \quad u = x^2 + 2, \; du = 2x \, dx
\]
\[
= -\frac{1}{2u} + C = -\frac{1}{2(x^2 + 2)} + C
\]
READY,
GET SET,...

2:30
\[ \int \sin^3 x \cos^2 x \, dx \]
INTEGRAL #2

\[
\int \sin^3 x \cos^2 x \, dx
\]

\[
= \int (1 - \cos^2 x) \cos^2 x \sin x \, dx
\]

\[
= -\int (1 - u^2)u^2 \, du \quad u = \cos x, \quad du = -\sin x \, dx
\]

\[
= -\int (u^2 - u^4) \, du = -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C
\]
INTEGRAL #3

READY, GET SET, ...

2:30
INTEGRAL #3

\[ \int \frac{(x + 2)^3}{x} \, dx \]

2:30
INTEGRAL #3

\[ \int \frac{(x + 2)^3}{x} \, dx \]

\[ = \int \frac{x^3 + 6x^2 + 12x + 8}{x} \, dx \]

\[ = \int \left( x^2 + 6x + 12 + \frac{8}{x} \right) \, dx \]

\[ = \frac{x^3}{3} + 3x^2 + 12x + 8 \ln|x| + C \]
READY, GET SET,...

2:30
INTEGRAL #4

\[ \int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}} \, dx \]
\[
\int e^x + e^{2x} + e^{3x} \frac{1}{e^{4x}} \, dx \\
= \int \left( \frac{e^x}{e^{4x}} + \frac{e^{2x}}{e^{4x}} + \frac{e^{3x}}{e^{4x}} \right) \, dx \\
= \int (e^{-3x} + e^{-2x} + e^{-x}) \, dx \\
= -\frac{1}{3}e^{-3x} - \frac{1}{2}e^{-2x} - e^{-x} + C
\]
INTEGRAL #5

READY,
GET SET,...

2:30
\[ \int \sqrt{x} \ln x \, dx \]
\[
\int \sqrt{x} \ln x \, dx
\]

\[
= \frac{2x^{3/2} \ln x}{3} - \frac{2}{3} \int x^{1/2} \, dx
\]

integrate by parts

\[
= \frac{2x^{3/2} \ln x}{3} - \frac{4x^{3/2}}{9} + C
\]
INTEGRAL #6

READY,
GET SET,...

2:30
\[ \int \frac{\sec^2 2x}{\sqrt[3]{2 + \tan 2x}} \, dx \]
\[ \int \frac{\sec^2 2x}{\sqrt[3]{2 + \tan 2x}} \, dx \]

\[ = \frac{1}{2} \int \frac{1}{\sqrt[3]{u}} \, du \quad u = 2 + \tan 2x, \quad du = 2 \sec^2 2x \, dx \]

\[ = \frac{3u^{2/3}}{4} + C \]

\[ = \frac{3(2 + \tan 2x)^{2/3}}{4} + C \]
INTEGRAL #7

READY,
GET SET,...

2:30
\[ \int (x + 17) \sqrt{x + 29} \, dx \]
\[
\int (x + 17) \sqrt{x + 29} \, dx
= \int (u - 12) \sqrt{u} \, du \quad u = x + 29, \; x = u - 29, \; dx = du
\]
\[
= \int \left( u^{3/2} - 12u^{1/2} \right) \, du = \frac{2u^{5/2}}{5} - 8u^{3/2} + C
\]
\[
= \frac{2}{5} (x + 29)^{5/2} - 8(x + 29)^{3/2} = \frac{2}{5} (x + 9)(x + 29)^{3/2} + C
\]
INTEGRAL #8

READY,
GET SET,…

2:30
INTEGRAL #8

\[ \int \frac{1 - x}{\sqrt{1 - x^2}} \, dx \]

2:30
INTEGRAL #8

\[ \int \frac{1 - x}{\sqrt{1 - x^2}} \, dx \]

\[ = \int \left( \frac{1}{\sqrt{1 - x^2}} - \frac{x}{\sqrt{1 - x^2}} \right) \, dx \]

\[ = \arcsin x + \sqrt{1 - x^2} + C \]
INTEGRAL #9

READY,
GET SET,…

2:30
\[ \int \frac{\sin x}{(1 - \sin x)(1 + \sin x)} \, dx \]
\[ \int \frac{\sin x}{(1 - \sin x)(1 + \sin x)} \, dx \]

\[ = \int \frac{\sin x}{1 - \sin^2 x} \, dx = \int \frac{\sin x}{\cos^2 x} \, dx \]

\[ = - \int \frac{1}{u^2} \quad u = \cos x, \quad du = - \sin x \, dx \]

\[ = \frac{1}{u} + C = \frac{1}{\cos x} + C = \sec x + C \]
INTEGRAL #10

READY, GET SET,...

2:30
INTEGRAL #10

\[ \int \frac{1}{x^2 \sqrt{1 - x^2}} \, dx \]
\[
\int \frac{1}{x^2 \sqrt{1 - x^2}} \, dx
\]

\[
= \int \frac{\cos \theta}{\sin^2 \theta \cdot \cos \theta} \, d\theta \quad \text{\(x = \sin \theta, \ \, dx = \cos \theta \, d\theta\)}
\]

\[
= \int \csc^2 \theta \, d\theta = -\cot \theta + C
\]

\[
= -\frac{\sqrt{1 - x^2}}{x} + C
\]
INTEGRAL #11

READY,
GET SET,...

2:30
\[ \int \frac{x - 1}{x^2(x + 1)} \, dx \]
\[
\int \frac{x - 1}{x^2(x + 1)} \, dx = \int \left( \frac{2}{x} - \frac{1}{x^2} - \frac{2}{x + 1} \right) \, dx \quad \text{partial fractions}
\]

\[
= 2 \ln|x| + \frac{1}{x} - 2 \ln|x + 1| + C
\]
INTEGRAL #12

READY, 
GET SET,...

2:30
INTEGRAL #12

\[
\int \frac{\sin x + \sec x}{\tan x} \, dx
\]
\[
\int \frac{\sin x + \sec x}{\tan x} \, dx
\]

\[
= \int \left( \frac{\sin x}{\tan x} + \frac{\sec x}{\tan x} \right) \, dx
\]

\[
= \int (\cos x + \csc x) \, dx
\]

\[
= \sin x + \ln|\csc x - \cot x| + C
\]
READY,
GET SET,...

2:30
INTEGRAL #13

\[ \int \frac{\ln x}{x + x(\ln x)^2} \, dx \]

2:30
INTEGRAL #13

\[
\int \frac{\ln x}{x + x(\ln x)^2} \, dx
= \int \frac{\ln x}{x(1 + (\ln x)^2)} \, dx
= \int \frac{u}{1 + u^2} \, du
\quad u = \ln x, \quad du = \frac{1}{x} \, dx
= \frac{1}{2} \ln(1 + u^2) + C
= \frac{1}{2} \ln(1 + (\ln x)^2) + C
\]