The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the Integration Champion!
INTEGRAL #1

READY,

GET SET,…

2:30
\[ \int \frac{x}{e^{x^2}} \, dx \]
INTEGRAL #1

\[ \int \frac{x}{e^{x^2}} \, dx \]

\[ = \int x e^{-x^2} \, dx \]

\[ = \frac{-1}{2} \int e^u \, du \quad [u = -x^2, \quad du = -2x \, dx] \]

\[ = -\frac{e^u}{2} + C = -\frac{e^{-x^2}}{2} + C \quad \text{or} \quad -\frac{1}{2e^{x^2}} + C \]
INTEGRAL #2

READY, GET SET, ...

2:30
\( \int \frac{x}{x^4 + 6x^2 + 9} \, dx \)
INTEGRAL #2

\[
\int \frac{x}{x^4 + 6x^2 + 9} \, dx
= \int \frac{x}{(x^2 + 3)^2} \, dx
= \frac{1}{2} \int \frac{1}{u^2} \, du \quad [u = x^2 + 3, \quad du = 2x \, dx]
= \frac{1}{2} \cdot \frac{-1}{u} + C = \frac{1}{2(x^2 + 3)} + C
\]
INTEGRAL #3

READY,
GET SET,...

2:30
\[ \int \frac{x + 4}{x^2 + 4} \, dx \]
INTEGRAL #3

\[
\int \frac{x + 4}{x^2 + 4} \, dx
= \int \left( \frac{x}{x^2 + 4} + \frac{4}{x^2 + 4} \right) \, dx
= \frac{1}{2} \ln(x^2 + 4) + 2 \arctan \frac{x}{2} + C
\]
INTEGRAL #4

READY,
GET SET,...

2:30
\[ \int \frac{\ln x}{\sqrt{x}} \, dx \]
INTEGRAL #4

\[
\int \frac{\ln x}{\sqrt{x}} \, dx
\]

\[
= 2\sqrt{x} \ln x - \int 2\sqrt{x} \cdot \frac{1}{x} \, dx
\]

\[
= 2\sqrt{x} \ln x - 2 \int \frac{1}{\sqrt{x}} \, dx
\]

\[
= 2\sqrt{x} \ln x - 4\sqrt{x} + C
\]

by parts: \( u = \ln x, \quad dv = \frac{dx}{\sqrt{x}} \)
INTEGRAL #5

READY,
GET SET,...

2:30
INTEGRAL #5

\[\int \frac{\sqrt{x}}{\sqrt{x\sqrt{x} + \sqrt{7}}} \, dx\]

2:30
INTEGRAL #5

\[ \int \frac{\sqrt{x}}{\sqrt{x\sqrt{x} + \sqrt{7}}} \, dx \]

\[ = \frac{2}{3} \int \frac{1}{\sqrt{u}} \, du \quad \left[ u = x\sqrt{x} + 7, \quad du = \frac{3}{2}\sqrt{x} \right] \]

\[ = \frac{2}{3} \cdot 2\sqrt{u} = \frac{4}{3} \sqrt{x\sqrt{x} + \sqrt{7}} \]
INTEGRAL #6

READY,
GET SET,...

2:30
\[\int \frac{x^2}{x^2 + 1} \, dx\]
\[ \int \frac{x^2}{x^2 + 1} \, dx \]

\[ = \int \frac{x^2 + 1 - 1}{x^2 + 1} \, dx = \int \left( \frac{x^2 + 1}{x^2 + 1} - \frac{1}{x^2 + 1} \right) \, dx \]

\[ = \int \left( 1 - \frac{1}{x^2 + 1} \right) \, dx \]

\[ = x - \arctan x + C \]
INTEGRAL #7

READY,
GET SET,...

2:30
\[ \int x \sec x \tan x \, dx \]
\[ \int x \sec x \tan x \, dx \]

\[ = x \sec x - \int \sec x \, dx \]

Integration by parts:
\[ u = x, \quad dv = \sec x \tan x \, dx \]
\[ du = dx, \quad v = \sec x \]

\[ = x \sec x - \ln|\sec x + \tan x| + C \]
READY, GET SET,...

2:30
\[ \int x \sqrt{x + 5} \, dx \]
INTEGRAL #8

\[
\int x \sqrt{x + 5} \, dx
\]

\[
= \int (u - 5) \sqrt{u} \, du \quad [u = x + 5, \quad du = dx]
\]

\[
= \int \left(u^{3/2} - 5u^{1/2}\right) \, du = \frac{2u^{5/2}}{5} - 5 \cdot \frac{2u^{3/2}}{3} + C
\]

\[
= \frac{2(x + 5)^{5/2}}{5} - \frac{10(x + 5)^{3/2}}{3} + C
\]
INTEGRAL #9

READY,
GET SET,...

2:30
INTEGRAL #9

\[ \int (\sec x + \tan x)^2 \, dx \]

2:30
INTEGRAL #9

\[
\int (\sec x + \tan x)^2 \, dx
\]

\[
= \int \left( \sec^2 x + 2 \sec x \tan x + \tan^2 x \right) \, dx
\]

\[
= \int \left( \sec^2 x + 2 \sec x \tan x + \sec^2 x - 1 \right) \, dx
\]

\[
= \int \left( 2 \sec^2 x + 2 \sec x \tan x - 1 \right) \, dx
\]

\[
= 2 \tan x + 2 \sec x - x + C
\]
INTEGRAL #10

READY,
GET SET,...

2:30
INTEGRAL #10

\[ \int \frac{dx}{x^2 + 2x + 10} \]
\[
\int \frac{dx}{x^2 + 2x + 10} = \int \frac{dx}{(x + 1)^2 + 9} = \int \frac{du}{u^2 + 3^2} \quad [u = x + 1, \quad du = dx] = \frac{1}{3} \text{arctan} \frac{u}{3} + C = \frac{1}{3} \text{arctan} \frac{x + 1}{3} + C
\]
READY,  
GET SET,...  

2:30
\[ \int \frac{dx}{\sqrt{x + 1} + \sqrt{x - 1}} \]
\[
\int \frac{dx}{\sqrt{x + 1} + \sqrt{x - 1}} \\
= \int \frac{1}{\sqrt{x + 1} + \sqrt{x - 1}} \cdot \frac{\sqrt{x + 1} - \sqrt{x - 1}}{\sqrt{x + 1} - \sqrt{x - 1}} \, dx \\
= \int \frac{\sqrt{x + 1} - \sqrt{x - 1}}{2} \, dx \\
= \frac{(x + 1)^{3/2} - (x - 1)^{3/2}}{3} + C
\]
INTEGRAL #12

READY, GET SET, ...

2:30
\[ \int \frac{1}{\chi} \, d\chi \]
\[
\int \arctan \frac{1}{x} \, dx
\]

\[
\begin{align*}
\text{integrate by parts:} & \quad u = \arctan \frac{1}{x}, \quad dv = dx \\
du &= -\frac{1}{x^2+1} \, dx, \quad v = x
\end{align*}
\]

\[
= x \arctan \frac{1}{x} + \int \frac{x}{x^2 + 1} \, dx
\]

\[
= x \arctan \frac{1}{x} + \frac{1}{2} \ln(x^2 + 1) + C
\]